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A STATISTICAL STUDY OF BREAKOUTS IN OIL-WELLS IN ALBERTA

by



CHRISTIAN KWAKU FORDJOR

A THESIS

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## ABSTRACT

Breakouts or preferred directional spalling along the walls of boreholes have been observed in many wells throughout the sedimentary basin in most parts of Alberta. Many studies have been carried out into the possible cause of this phenomenon. In this study, data focussing on the relationship of breakouts with depth, rock types and age of the rocks have been investigated in 50 wells scattered through most parts of Alberta plus one well in British Columbia. A regression of breakout azimuths on depths shows regression coefficients not significantly different from zero. Furthermore, it has been noted that breakouts have no relation with the rock types and the age of the rocks. The breakout azimuths in a well are tightly grouped around a mean direction which, in most cases, is NW-SE.

These observations and other evidences lead to the conclusion that breakout formations are better explained by the stress concentration at the walls of the boreholes, rather than the result of the drill encountering pre-existing fractures or zones of weaknesses. From this conclusion, the orientation of the smaller horizontal principal stress is NW-SE and the larger horizontal principal stress is NE-SW. The consistency of the breakout azimuths with depth, wherever these phenomena are observed, may imply that such large horizontal principal stresses are not necessarily limited to the top crustal sedimentary rocks but may be continued down into the deep and older





Precambrian igneous rocks of the Earth's crust.





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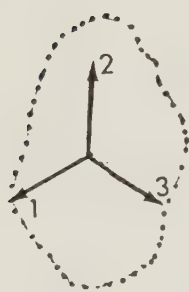
## I. THE PHENOMENON OF ALIGNED BREAKOUTS AND ITS MEANING

The primary purpose of borehole dipmeter surveys is to determine magnitudes and azimuths of dips of bedding planes from observations in a single hole. Prior to such surveys, the subsurface structural picture was determined primarily from seismic sections and correlation of marker horizons between three or more wells not drilled on a straight line. Unfortunately, geophysical surveys normally cover large areas and are good for determination of average regional dips but give poor resolution of local dip variations. In a like manner dip determination of marker horizons from correlation is valid only if the bed used is truly a plane surface and continuous between wells.

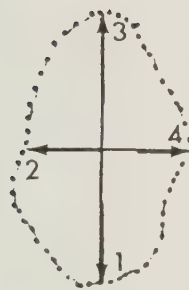
About 1970 the **Four Arm Dipmeter** (also known as **The High Resolution Dipmeter, HDT**) began to supersede the three arm type of tool. This thesis is largely concerned with oil-wells of non-circular section. Figure 1 reveals that the HDT is capable of determining two different diameters of a non-circular hole and their azimuths, whereas the three arm instrument cannot do this.

In the four arm dipmeter shown in Figure 2, the four arms are azimuthally spaced  $90^\circ$  apart. Each spring loaded arm carries a pad of electrodes which enables the electrical resistivity of the rock near the electrodes to be recorded. The four correlation curves in Figure 3 show typical resistivity variations which enable beds to be recognised and correlated. A plane can be fitted to the four depth





**3 ARM**



**4 ARM**

Figure 1.... Possible positions assumed by 3-arm caliper and 4-arm dual calipers in the same elongated hole. (schematic)





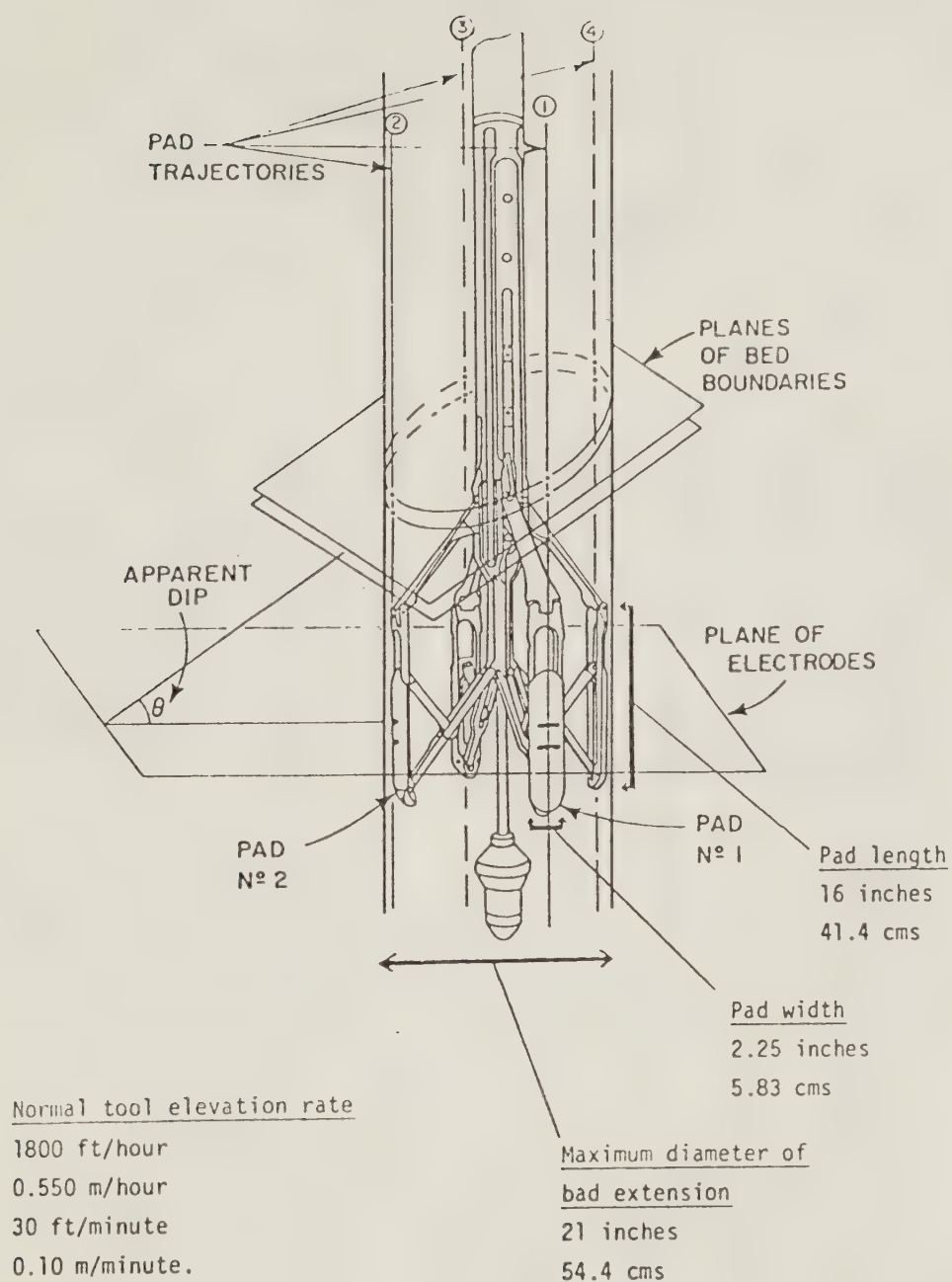


Figure 2.... Schlumberger 4-arm dipmeter tool.(courtesy of J. Cox)





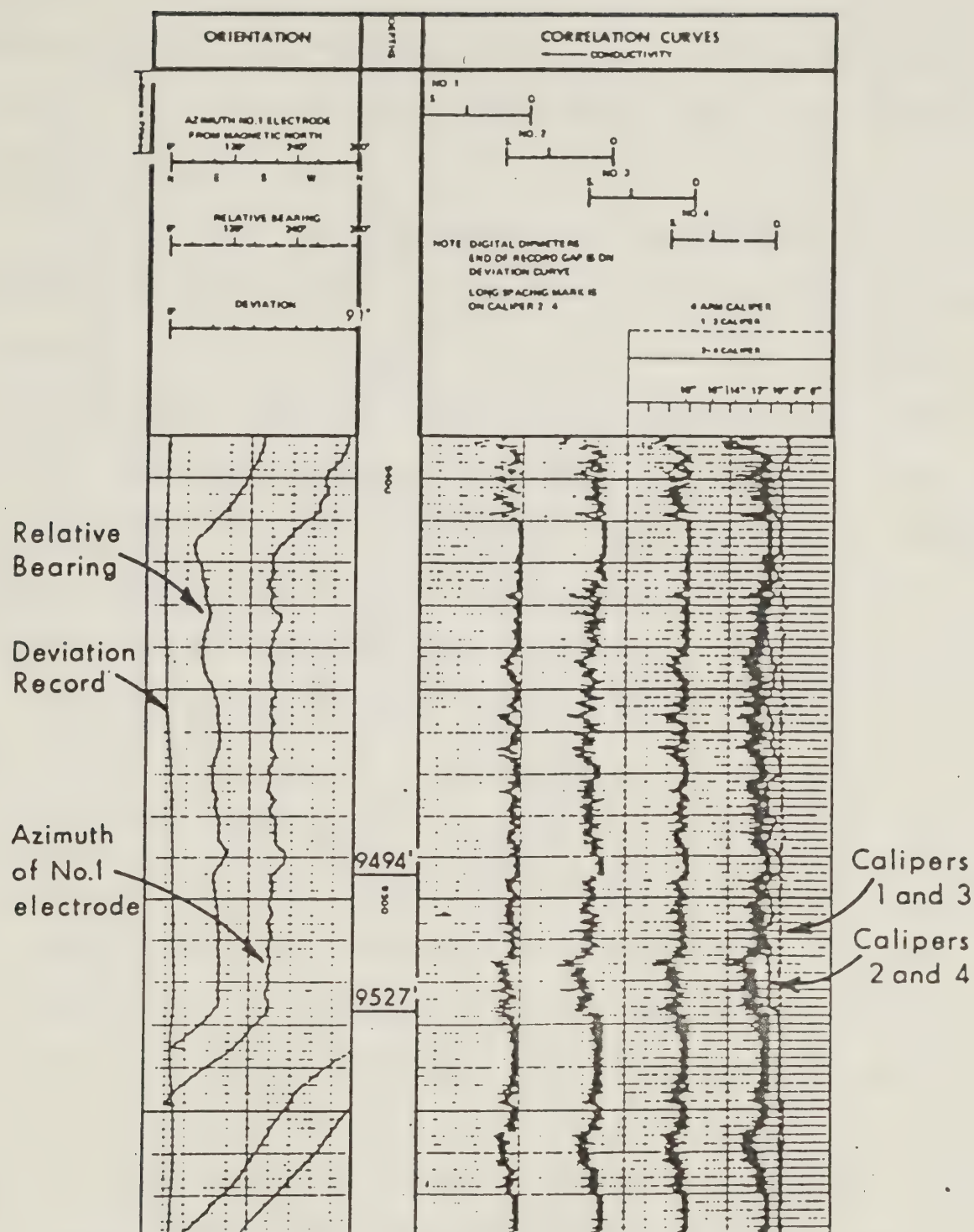


Figure 3.... Typical 4-arm dipmeter log record



values for a bed to give its dip in magnitude and direction. Resistivity changes on only one or two electrode pads may assist in identifying fractures as is illustrated schematically in figure 4.

Traces not yet discussed in figure 3 are of principal interest in this study. On the far right the two orthogonal diameters indicated by opposed pairs of calipers are recorded. These provide information concerning the shape of the borehole. On the left we require the trace which shows the azimuth of No.1 electrode, relative to a magnetic compass housed in the tool. Other traces on the left are;

1. The nearly vertical line which shows the deviation from verticality on a common scale from  $0^{\circ}$  to  $9^{\circ}$ .
2. A dashed line which runs diagonally or rotates as the tool is drawn up in the circular portions of the hole. This line ceases to rotate on encountering an elliptical zone and gives the relative bearing of the tool on a scale of 0 to 360 degrees with respect to azimuth of No.1 electrode. From this the bearing of the hole drift can be calculated.

The tool normally rotates as it is drawn up the well and in many depth ranges the calipers will indicate essentially equal diameters equal to the drill-bit diameter. This can be seen in the bottom and top sections of figure 3. It is observed, however, in other depth ranges that the tool ceases to rotate and the calipers indicate different diameters. This is the result of extensive fracturing





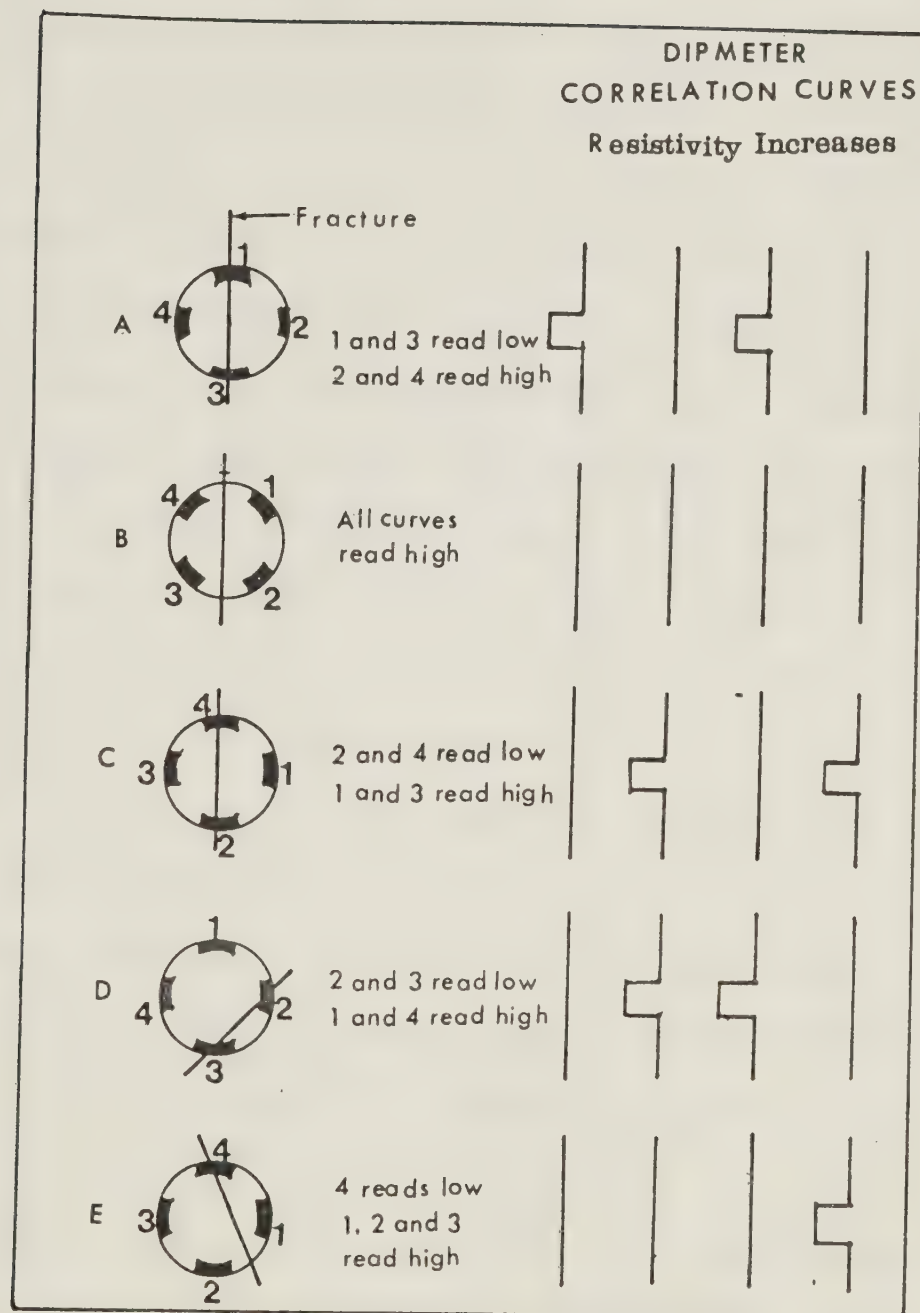


Figure 4..... Responses of HDT correlation curves to vertical fractures. (schematic)



causing the borehole to spall and wash out on opposite sides, thus becoming elliptical. Usually the smaller diameter is equal to the drill bit. This condition has been named by Babcock (1978) a **BREAKOUT**, and is illustrated in figure 5 D. Other situations encountered in a borehole are also shown in figures 5 A to C. In Figure 5 D one pair of calipers has become trapped in a locally elongated section of the hole. The orientation of the breakout can be estimated from the azimuth of the trapped caliper pads. In Figure 3, a breakout is shown between depths 9418 and 9527 feet. Calipers 2 and 4 (solid diameter line) indicate the greater diameter and the stationary azimuth trace shows No.1 electrode near  $200^{\circ}$ . The azimuth of the long axis of this breakout is near  $200-90=110^{\circ}$  relative to the magnetic north. The final orientation is  $110^{\circ}$  plus the magnetic declination.

The success of the HDT in determining sedimentary bedding planes has led to an extensive use of this tool in many areas, starting from the West Texas Ellenberger play in 1969, the cretaceous of North Louisiana and Mississippi and also in the limestone of the Mooringsport of the Waveland field of Mississippi.

During the interpretation of Schlumberger 4-arm dipmeter logs wells in Alberta, Cox (1970) observed that through the Wapiabi shale (a predominantly dark gray shale of both Colorado and post Colorado age in the upper cretaceous period) interval of the Strachen-Ricinus area (see Fig. 16 shaded portion) of West Central Alberta





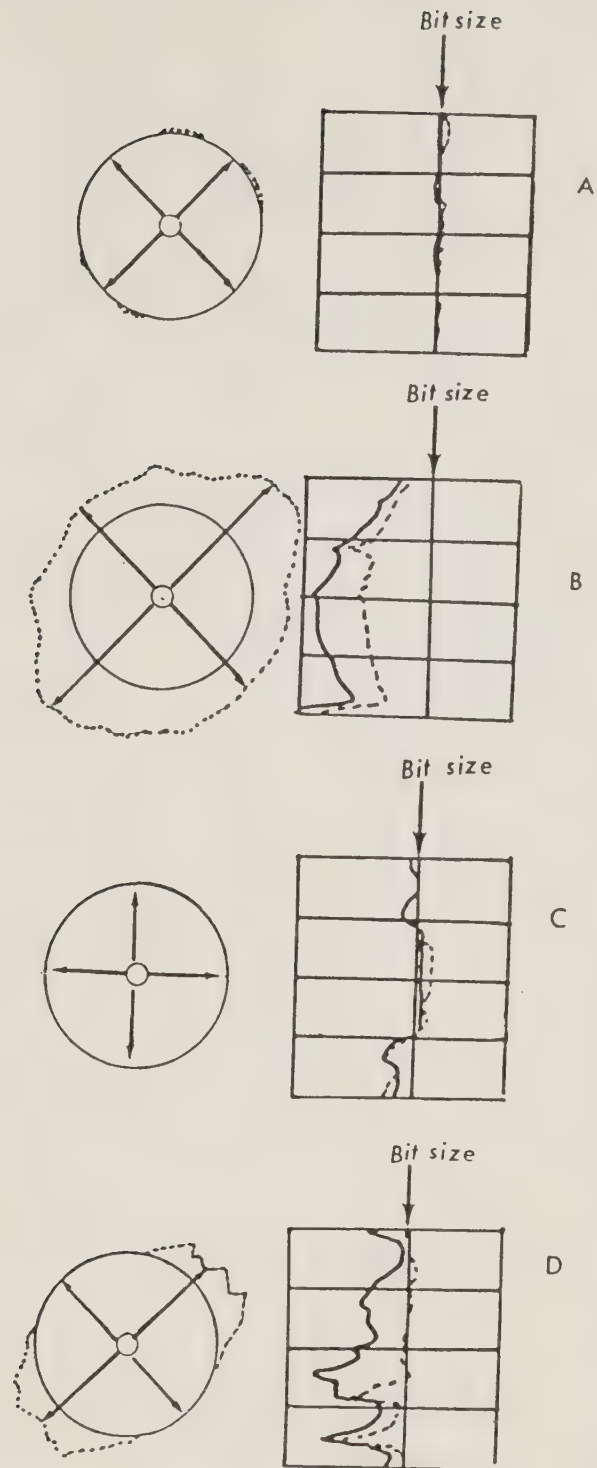


Figure 5.... Dual-caliper response to possible conditions of borehole shape.



adjacent to the Rocky Mountain foothills, where the borehole was not at bit size it was always elongated in the northwest-southeast direction. Structural dips were usually northeast or southwest. Nevertheless, several wells in this area in which structural dips were differently oriented showed the same northwest-southeast elongation. This observation led Cox to make a broader statistical study of 4-arm dipmeter logs in 31 zones from 17 wells scattered about Alberta and the North West Territories.

This new study ranged from Cretaceous shales to Devonian carbonates, in which dip values ranged from  $0^{\circ}$  to  $30^{\circ}$ . The orientation of the long axes of the holes had an average direction of  $N47^{\circ}W$  to  $S47^{\circ}E$ . A few elongations formed another group perpendicular to this major trend (Fig. 6 A). That the dip direction is not a controlling factor for the hole elongation is shown in figure 6 B.

An interesting feature of these elongations observed by Cox (1970) was that they were confined to discrete depth intervals within a hole and separated by uncaved beds. This observation led Babcock (1978) to suggest that the elongation might be caused by the drill encountering zones of steeply dipping fractures. Babcock later undertook a study of the phenomenon of hole elongation using the caliper portion of the 4-arm dipmeter logs, and extending his investigation to 23 wells in various sedimentary rocks widely distributed through Alberta.





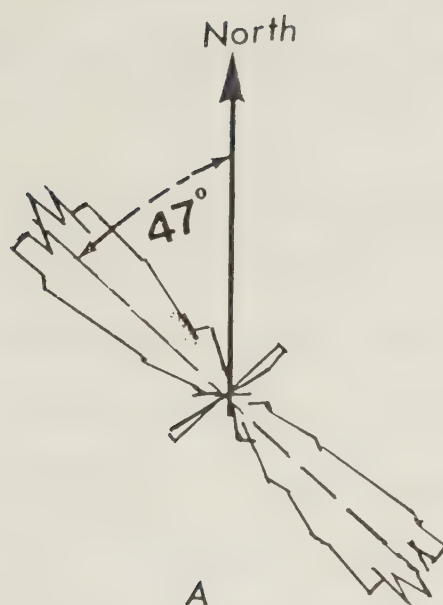


Figure 6.... Orientation of breakout long axis with respect to (A) North and (B) Dip azimuth. (J. Cox, 1970)



He noted the following properties of breakouts

1. The breakout zones may be short discrete events, or may persist over depth intervals of several tens or hundreds of feet.
2. A breakout of 2 to 4 feet commonly slows down the tool rotation or may have no noticeable effect on rotation; whereas a longer breakout is commonly associated with a cessation in the rotation of the tool.

To explain his observations, which were consistent with Cox's (1970) observations, Babcock turned to his study of the regional jointing in exposed bedrock of the Alberta plains and in the Fort McMurray area of northeast Alberta (Babcock, 1973, 1974, 1975). Within this area two orthogonal joint systems made up of vertically dipping extensional fractures are present. System 1 has sets striking northeast and northwest roughly normal to and parallel with the Rocky Mountain Belt. The northeast striking set is dominant in the sense that it is most commonly developed in that joints of this set usually cut across joints of other sets, which terminate against them. A second joint system (System 2, Babcock, 1973) having sets striking roughly North-South and East-West was also observed. This system was common in the Fort McMurray area and in the southernmost part of Alberta and occurred elsewhere in the province. These two joint systems were reported (Babcock, 1973) in nearly flat-lying sedimentary rocks ranging from late Devonian to Paleocene in age, in outcrops of shales, siltstone, sandstone, limestone



dolomite and coal distributed through many thousands of square kilometers and several thousands of meters of the stratigraphic column. Out of a total of 1,236 nearly vertically dipping joints measured, Babcock observed that although joint sets were well defined at most stations, set directions between stations varied considerably. Nevertheless, the sets striking northeast and northwest were most developed.

Regions covered by Babcock in his study of breakouts included the Claresholm district. Here he compared the joint azimuths measured at 10 outcrops and the azimuths of elongations from five wells. The mean directions of the elongations he observed showed a variation in strike ranging from  $129^{\circ}$  to  $155.5^{\circ}$ , thus corresponding closely to the northwest-southeast striking joint set. However, the elongations showed much less variability than do the strikes of individual joints of the northwest striking set in the area. A comparison of all azimuths of elongation (41 in all) with that of all joint strikes (1,236) in the area is shown in Figure 7. Clearly there is no close correspondence as a result of the predominance of one joint set striking at approximately  $175^{\circ}$  (North-South), which falls under Babcock's System 2.

Similar comparisons were made by Babcock in his Red Deer study between 400 azimuths of joints in bedrocks and 67 breakout elongations in nearby oil wells. Here, however, joint sets belong to the System 1 and the dominant direction





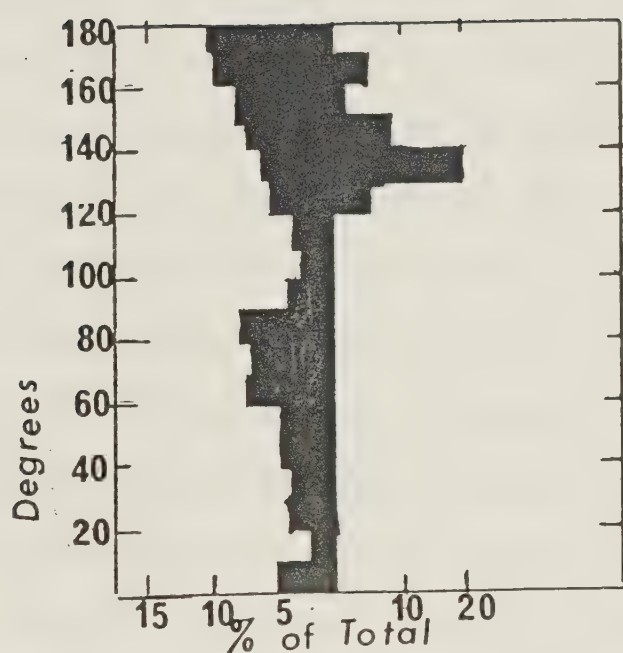


Figure 7.... Histogram comparing (left) strikes of 1,236 joints measured at 10 outcrops in Claresholm region, southwest Alberta, with (right) azimuths of 41 elongated breakouts from five wells. (Babcock, 1978)



of hole elongation was northwest-southeast at about 140 °.

Babcock's study of 23 wells scattered throughout Alberta led him to draw the following conclusions, some of which had been made by Cox (1970):

1. In most of the 23 wells studied, the breakouts are in the northwesterly direction. However, in some wells a small number of breakouts may follow a northeast trend.
2. The azimuths of elongation are parallel to well developed sets of regional joints which are present wherever joint directions have been studied.
3. Hole deviations from verticality, changes in the magnitudes and directions of bedding dips, and lithology are unrelated to breakouts.
4. The preferred orientation of breakouts could not be the result of the drill encountering vugs because this would lead to hole elongations with no preferred orientation.
5. Babcock further stated that through his informal discussion with engineers involved with hydraulic fracturing and from consideration of the tectonic setting, the inferred direction of intermediate stress in rocks of the Alberta plains is normal to the trend of the Rocky Mountain Belt or roughly Northeast Southwest and the maximum stress is vertical, with the minimum stress direction inferred to be Northwest Southeast parallel with the dominant azimuth of hole elongation.

From these general conclusions Babcock finally concluded that the breakouts occur as a result of the drill





encountering steeply dipping fractures or zones of steeply dipping fractures, which may or may not be open. The oversize holes in carbonate rocks he related to solution-widened joints.

Bell and Gough (1979) found Babcock's hypothesis for the cause of breakouts very difficult to accept. Their most serious objection was the fact that the azimuthal distribution of breakouts is very unlike that of joint sets at the surface. They argued that given four concentrations of joint directions on the surface (Babcock 1973) NW, NE, N, and E, they would expect concentrations of breakout azimuths in all the four directions on Babcock's hypothesis, rather than only one significant concentration of subsurface breakout direction (NW).

Secondly, Bell and Gough wondered if such surface joint directions would necessarily be parallel to those at depths extending beyond 2 km at which breakouts have been observed. In this connection they remarked that sediments involved in the study in the Alberta Plains were laid down in varying tectonic settings between Devonian and Cretaceous times; and furthermore joints may arise in several ways after deposition of sedimentary rocks.

In an attempt to find an interpretation that would account for both the breakouts in the wells and the surface joints System 1 of Babcock, Bell and Gough (1979) proposed that both features result from a general stress field acting throughout the Alberta Plains and oriented with the larger



principal horizontal stress NE-SW and the smaller of the two horizontal stresses parallel to the Rocky Mountain fold axes. Using the Kirsch equations for the stress near a circular hole, in a medium under biaxial stress, they argued that with large enough, unequal horizontal stresses oriented as above, the holes themselves could concentrate the stresses so as to produce subsurface breakouts with long axes in the NW-SE direction. Such concentrations of stress are believed to arise at the time of drilling.

If the breakouts in Alberta are shear fractures caused by the concentration of stress at the walls of the borehole as proposed by Bell and Gough (1979), the larger horizontal compression is orthogonal to the azimuth of the breakouts and hence nearly Northeast-Southwest in Alberta. Their hypothesis provides no indication of the magnitudes of the principal stresses other than the fact that the horizontal principal stresses are large and unequal. Gough and Bell (1982) have recently made a quantitative study of shear fracturing near a borehole, and have shown that such fractures can form breakouts with the observed properties.

Stress measurements in mines in several continents reveal that the vertical principal stress approximates (within 20% or less in most cases) the overburden pressure (McGarr and Gay, 1978). Furthermore, the proximity of the thrust faults of the Rockies to areas covered in the studies by Cox (1970) and Babcock (1978) coupled with the fact that the Rockies are fold mountains led Bell and Gough (1979) to



suggest that a situation with  $S_1$  (the maximum principal stress) vertical ( a normal-fault stress field) is most unlikely in Alberta.

This fact is further strengthened by the occurrence of breakouts at depths of 500 meters or less. Laboratory experiments by Gay (1973) indicate that the threshold pressure for spalling in a circular hole under equal orthogonal pressures is about 100 MPa. With unequal pressures, however, spalling would be expected with the larger compression around 30 MPa. But at a depth of 500 m the overburden pressure will be about 12 MPa. If  $S_1$  is vertical then both  $S_2$  and  $S_3$ , which are horizontal, will be smaller than this value (12 MPa) and may be too small to produce shear fractures. This now leaves us with either a thrust fault or a strike-slip (wrench) fault stress field. Bell and Gough's hypothesis fits either of these two stress fields and the breakouts are consistent with either case. It is worth noting that the breakouts alone cannot reveal which of the two stress situations prevails in Alberta.

In a later paper, Gough and Bell (1981) added additional wells in Alberta to their study of breakouts. They also extended the area covered into northeastern British Columbia. In addition results from hydraulic fracturing in an oil field in West-Central Alberta were discussed. They suggested that some local anomalies in breakout orientations might represent local variations in the stress field.





Lo and Morton (1976) attributed failures of tunnel roofs in Ontario to interaction of the tunnel with large horizontal compressive stresses transverse to the tunnel. Dusseault (1977) suggested that unequal horizontal compressive stresses prevail in several areas in Alberta, to account for the propagation of vertical fractures during hydraulic fracturing in Northeast-Southwest vertical planes. Such fractures imply a horizontal least principal stress ( $S_3$ ) directed Northwest-Southeast. The formation of tensile fractures normal to  $S_3$  during hydraulic fracturing was first proposed by Hubbert and Willis (1957). This leads to the expectation that in a thrust stress field ( $S_3$  vertical) hydraulically formed fractures will be horizontal (Kehle, 1964). This is difficult to verify in many cases, because horizontal fractures are hard to detect. Zoback et al (1977) argue that the inflatable packers inhibit such horizontal fractures, and that horizontal fractures form only if fluid penetrates pre-existing planes of weakness. This contention is supported by laboratory experiments by Haimson and Fairhurst (1970). Haimson (1976b) suggested that if  $S_v = S_3$ , then the induced fractures initiate in a vertical plane and then become horizontal as they propagate based on the fact that energy propagates along the path of least resistance. This can be observed in the pressure-time history where the asymptotic shut-in pressure is smaller than the instantaneous shut-in pressure. In such a situation the instantaneous shut-in pressure is not equal to the least



principal stress but rather the smaller horizontal principal stress with the asymptotic shut-in pressure being equal to the least principal stress  $S_3$ .

In the J Lease of the Pembina Oilfield, Macleod (1977) noted during secondary oil recovery from Cardium Formation sandstone that permeability was greatest in the Northeast-Southwest direction. This observation implied either a fracture system or else a permeability trend oriented Northeast-Southwest. But Neilson (1957) showed the isopach axes in this formation and area to be trending Northwest-Southeast. Thus one would expect a permeability trend related to the process of deposition to run NW-SE. Gough and Bell (1981) therefore concluded that Macleod's observation could be better explained in terms of vertical fractures oriented NE-SW, which would imply a NW-SE orientation of  $S_3$ . Six wells logged with the HDT in this area were identified and studied by Gough and Bell. The breakouts occur in rocks of Paleozoic ages over depths from 2500 to 4500 meters. Azimuths of hole elongation were tightly grouped with mean values ranging between  $132^\circ$  and  $141^\circ$ , with variations over approximately  $20^\circ$  in individual wells.

Imperial Oil in 1978 published information on their Cold Lake projects in far eastern Alberta. Induced vertical fracture orientations varied from  $N\ 30^\circ\ E$  to  $N\ 45^\circ\ E$  implying that the minimum principal stress was horizontal in this region and oriented between  $120^\circ$  and  $135^\circ$ . The larger





horizontal stress would then lie between  $30^{\circ}$  and  $45^{\circ}$ . Imperial Oil ran no HDT logs in wells in the Cold Lake region (J.S.Bell, personal communication with the Company). However, Gough and Bell (1981) located one well in this region logged by the HDT. The azimuth of a single breakout, at depth 477 to 479 m in the Clearwater Formation, was  $131^{\circ}$  suggesting the major horizontal stress direction of  $41^{\circ}$ . Recently, Gough and Bell (1982) have reported comparisons of breakout azimuths in Colorado, in East Texas and in northwestern Canada with other indications of stress orientations in those regions. In every case the observations support their explanation of breakouts as a consequence of stress concentration by the borehole in an anisotropic stress field. The hydraulically induced fractures at West Pembina and at Cold Lake indicate a strike-slip type stress field ( $S_1$  and  $S_2$  horizontal,  $S_3$  vertical) is most likely present in most parts of Alberta.

This thesis reports a study of the statistics of the azimuthal distribution of breakouts in oil-wells in Alberta in relation to depth, lithology, formation and age of rocks, together with the geophysical implications of breakouts.



## II. STRESS AND FRACTURE IN THE EARTH'S CRUST

### STRESS

If one considers a simple prism of cross-sectional area  $a$  subjected to a force  $F$ , as shown in Fig. 8, the stress  $S_z$  acting on the end surface ABCD is given by

$$S_z = F/a \text{ -----2-1}$$

It can be seen that the force  $F$  has no component acting parallel to the ABCD surface. This means it exerts no shear stress on this surface. By definition, any stress acting perpendicular to a surface along which the shear stress is zero is a principal stress. Thus in this case  $S_z$  is a principal stress. Here, the suffix indicates the direction in which the stress acts. Other principal stresses may be orientated parallel to the X- and Y- axes, and would be designated  $S_x$  and  $S_y$  respectively. If the relative intensities of the principal stresses are known, they may be termed the maximum (or greatest), intermediate and minimum (or least) principal stresses (i.e.  $S_1$ ,  $S_2$ ,  $S_3$  respectively).

If one considers the action of the force  $F$  on a surface GHIJ inclined at an angle  $\phi$  as shown in the Figure 8, the component of  $F$  acting normal to GHIJ is given by,

$$F_n = F \cdot \sin \phi.$$

However, it will be seen that the area  $a'$  of GHIJ is greater than the area  $a$  of ABCD, and that

$$a' = a / \sin \phi.$$

Therefore, the normal stress  $S_n$  acting on the inclined



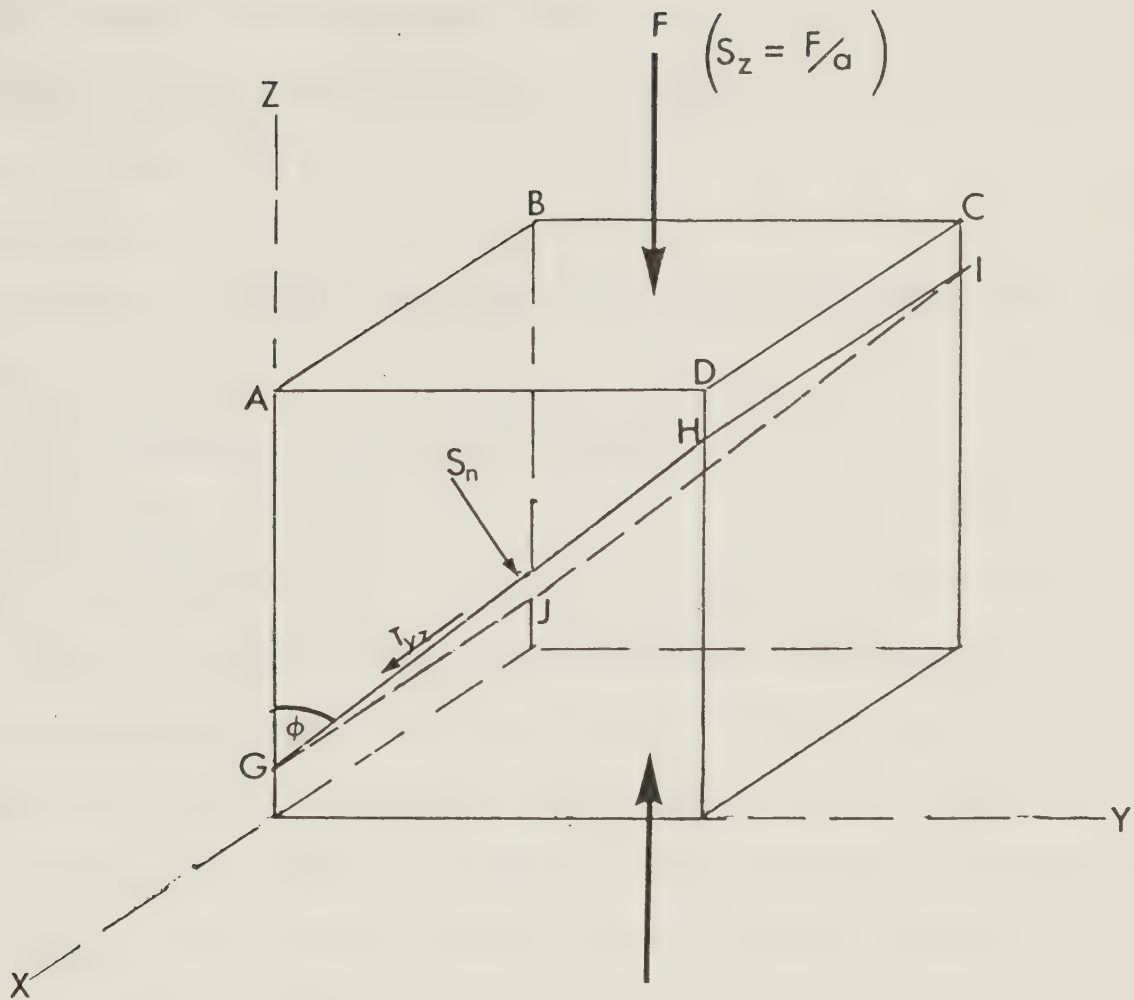


Figure 8.... Normal and Shear stresses acting on external and internal surfaces of a unit cube subjected to a force  $F$





surface is

$$F_n / a' = F \cdot \sin \theta / a' = (F/a) \cdot \sin^2 \theta$$

so that

$$S_n = S_z \cdot \sin^2 \theta \text{ -----2-2}$$

Similarly the component of  $F$  tangential to the inclined plane is given by

$$F_t = F \cdot \cos \theta.$$

Consequently the shear stress ( $T$ ) acting along the plane equals

$$F_t / a' = F \cdot \cos \theta / a' = (F/a) \cdot \cos \theta \cdot \sin \theta$$

that is

$$T = S_z \cdot \cos \theta \cdot \sin \theta \text{ -----2-3}$$

Shear stresses as well as other stresses may be related to co-ordinate axes. We denote by  $T_{zy}$  the shear stress on  $yz$ -planes acting parallel to the  $y$ -axis. Similarly  $T_{zx}$  denotes a shear stress acting in the  $zx$ -planes parallel to the  $x$ -axis. The stress across a plane whose normal is in the  $y$ -direction will have components  $T_{yx}$ ,  $S_y$ ,  $T_{yz}$  and that across a plane whose normal is in the  $x$ -direction will have components  $S_x$ ,  $T_{xy}$ ,  $T_{xz}$ . The nine quantities below

$$\begin{array}{ccc} S_x & T_{xy} & T_{xz} \\ T_{yx} & S_y & T_{yz} \\ T_{zx} & T_{zy} & S_z \end{array} \text{ -----2-4}$$

are called the stress components and give a complete specification of the stress at a point. It can be shown that  $T_{yx} = T_{xy}$ ,  $T_{yz} = T_{zy}$  and  $T_{zx} = T_{xz}$ , so that only six quantities out of the nine are needed to specify the stress at a point.



The stress components in Eqn.(2-4) are in fact the components of a mathematical entity called a tensor,(and tensor analysis is much used in developing the higher parts of the theory of elasticity where  $x,y,z$ , are replaced by  $1,2,3$ , respectively). A tensor in which  $T_{xy} = T_{yx}$  ,  $T_{yz} = T_{zy}$  ,  $T_{xz} = T_{zx}$  is said to be symmetric.

In a biaxial stress field it can be inferred from Eqn. (2-2) and Eqn. (2-3) that the stresses acting normally and tangentially to a plane inclined at  $\phi$  to  $S_1$  and hence  $(90+\phi)$  to  $S_3$  will be represented (on superposition) by

$$S_n = S_1 \cdot \sin^2 \phi + S_3 \cdot \cos^2 \phi$$

and

$$T = (S_1 - S_3) \cdot \sin \phi \cdot \cos \phi \text{-----2-5}$$

which can be written in terms of the double angle  $2\phi$  as

$$S_n = (S_1 + S_3)/2 - ((S_1 - S_3) \cdot \cos 2\phi)/2 \text{ and}$$

$$T = ((S_1 - S_3) \cdot \sin 2\phi)/2 \text{-----2-6}$$

Equation 2-6 may be represented graphically by means of Mohr's stress circle, which will be discussed later in this chapter.

Subsurface rocks are normally in a state of compressive stress except very near the surface because of the weight of overlying rocks. This overburden weight creates stresses in both the vertical and horizontal directions. The stress field at a point can be represented by  $S_1$ ,  $S_2$ ,  $S_3$  as already defined. Over long periods of geologic time the earth has exhibited an appreciable degree of mobility during which rocks have been repeatedly stressed to the limit of failure





to produce faulting and folding. The problem of rock squeeze (i.e. continuous deformation of the rock) has been recognised in parts of the crust. In Southern Ontario structures built in rocks have exhibited distress in various degrees during or subsequent to construction (Lo, 1978). Geological processes such as faulting, folding, and pop-ups or buckling of the surface rock strata, without any apparent change of external loading, may be interpreted as evidence of existence of high horizontal stresses. In a study of geological features and movements of structures in rock in New York State, Rose (1951) associated the rock squeeze with the possible existence of high horizontal in-situ stress in the rock formations. Such phenomena require that substantial differences must exist between the principal stresses. Sbar and Sykes (1973) have used information from postglacial geologic features such as east-west rock squeeze in western New York and Niagara, and bridge abutments moving together to infer that the maximum compressive stress in this area has an easterly trend. Postglacial buckles or pop-ups near Chippewa Bay, New York have also been ascribed by Sbar and Sykes as not due to any environmental factors other than large horizontal compressive stresses. Pop-ups have also been encountered elsewhere in western New York where the lithostatic load has been reduced by quarrying (Sbar and Sykes personal commun. with J. Davies, 1972). Deformation indicative of a high horizontal compressive stress was also observed in various parts of Ontario (Coates, 1964), where a



pop-up about two and one half meters high, striking northwest, occurred overnight in a quarry after it had been excavated to a depth of 15 meters. Finally the results obtained from geologic features, in-situ stress measurements and earthquake distributions in eastern and central North America may be related to the presence of high stress.

The orientation of the trajectories of the principal stresses in space is largely determined by the condition which they must satisfy at the earth's surface. This is a free surface, on which normal and shear stresses vanish. Since the only planes on which the shear stresses are zero are those perpendicular to the principal stresses, it follows that one of the three trajectories of principal stress must end perpendicular to the surface of the ground, and the other two must be parallel to this surface. Therefore, in regions of gentle topography with simple geologic structures the principal stresses should be nearly horizontal and vertical, with the vertical stress approximately equal to the pressure of overlying material. Measurements of stress in mines in several continents show that in many cases the vertical normal stress is, within about 20%, equal to the overburden pressure  $S_v = \rho \cdot g \cdot h$ , where  $\rho$  is the density of the material,  $g$  is gravity and  $h$  is the depth. If the lateral stress were due solely to elastic response of the rock  $S_v$ , the horizontal principal stresses would be  $S = s = \nu / (1 - \nu) \cdot S_v = S_v / 3$  where  $\nu$  is Poisson's ratio. Such a stress field would produce normal



faults at any depth at which the stress difference exceeded the strength of the rock. However, observations in underground structures, quarries and hydraulic fracturing in boreholes indicate that the horizontal stresses  $S$  and  $s$   $S > s$  are not generally equal and do not vanish close to the surface, a necessary consequence if the horizontal stresses were due simply to elastic response of the rock to overburden pressure.

Measurements in several continents of in-situ stress in rocks show that in many cases the horizontal stresses are much higher than the vertical stress calculated from the overlying weight of the rock (McGarr and Gay, 1978 ; Hast, 1973; Herget, 1974). Frequently the horizontal stresses  $S$  and  $s$  increase linearly with depth. It is thus evident that large and unequal horizontal principal stresses are widespread in the continental crust of the earth.

### Relationships Between Principal Stresses at Failure

Most substances behave elastically at low stresses. As the stress is increased the body will begin to yield at some point if it is ductile, while if the body is brittle it will fracture at some point without appreciable yielding. The term fracture implies the appearance of distinct surfaces of separation in the body and yield is used for the onset of plastic deformation. Flow is used for unrestricted plastic deformation. Both flow and fracture (also known as failure) are observed on large and small scales in geologic material so that the criteria for them are vital for the





interpretation of geologic phenomena.

For brittle materials in tension, *tensile or cleavage* fracture takes place across a surface perpendicular to the direction of tension (Fig. 9 A). For brittle materials in compression, shear fracture takes place along a pair of planes or a cone approximately in the direction of the greatest shear stress but always between this direction and the direction of the largest compressive stress ( Fig. 9 B). The maximum shear stress theory, which dates back to Coulomb (1773), states that failure occurs at a point when the maximum shear stress is equal to the shear strength of the material. If  $S_1 \geq S_2 \geq S_3$  are the principal stresses at a point, taking compressive stress positive, the maximum shear stress has magnitude  $(S_1 - S_3)/2$  , and occurs across a plane containing the direction of  $S_2$  and whose normal bisects the angle between the greatest and least principal stresses. The theory implies that if  $C_0$  is the compressive strength of the material in an unconfined compression test in which  $S_3 = S_2 = 0$ ,  $S_1 = C_0$ , the material will fail across any plane inclined at  $45^\circ$  to the direction of compression for a frictionless material. For real materials the angle is less than  $45^\circ$ . The theory also implies that the tensile and compressive strengths are equal. The theory has been modified by Navier to fit qualitatively most of the observed facts, and is used by Anderson in a discussion of the types of geological faulting. In this modification, instead of assuming that fracture takes place across the plane over which the shear



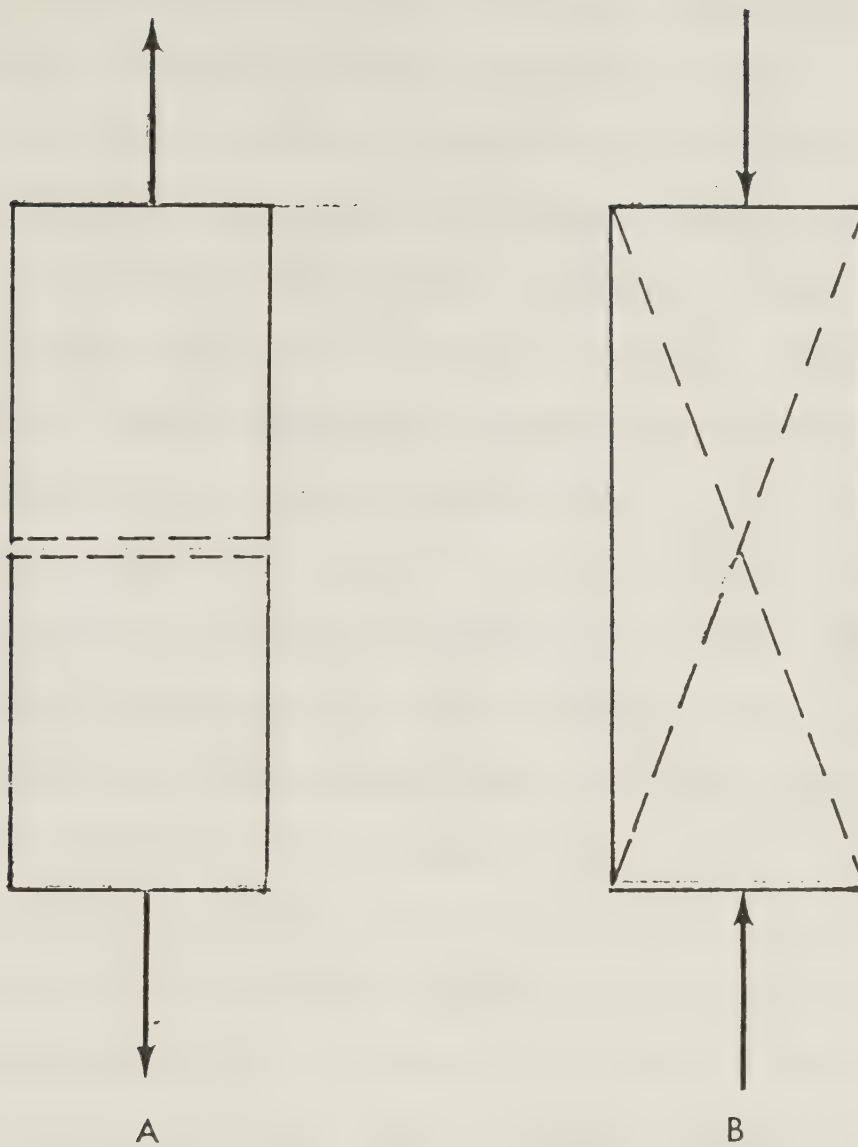


Figure 9.... Failure of a brittle material.(A) Tensile fracture and (B) Shear fracture



stress first becomes equal to the shear strength  $S_0$  of the medium, it assumes that this shear strength is increased by a constant  $u$  times the normal pressure across the plane. Because of the analogy to ordinary friction in which the tangential force is  $u$  times the normal reaction,  $u$  is known as the coefficient of internal friction. Thus, if  $S_n$  and  $T$  are the normal and shear stresses across a plane, fracture takes place on the plane at which the magnitude of  $T$  first becomes equal to  $S_0 + u.S_n$ . Therefore,

$$|T| = S_0 + u.S_n \text{ -----2-7}$$

This is the Navier-Coulomb criterion for shear failure. Let us discuss the case of two dimensions first. If  $S_1$ , and  $S_2$  are the principal stresses, we can then write Eqn. (2-6) as

$$S_n = (S_1 + S_2)/2 + (S_1 - S_2)\cos 2\phi/2$$

$$T = (S_1 - S_2).\sin 2\phi/2 \text{ -----2-8}$$

Only values of  $\phi$  between 0 and  $90^\circ$  need be considered, since only the magnitude of  $T$  occurs in Eqn. (2-7), so that changing the sign of  $\phi$  only changes the sign of  $\sin 2\phi$  and does not affect the magnitude of  $S_n$  or  $T$ . The shear stress at failure is thus symmetrical about  $\phi=0$ . It can be shown that failure takes place across a plane whose normal makes  $\phi$  with  $S_1$  where  $\tan 2\phi = 1/u$ . If  $u=0$ , then  $\phi=45^\circ$ ; if  $u=1$ ,  $2\phi=135^\circ$  and  $\phi=67.5^\circ$ ; if  $u \rightarrow \infty$ ,  $\phi \rightarrow 90^\circ$  that is as  $u$  increases the plane of fracture moves towards the direction of maximum stress. Values of  $u$  of the order of 1 are inferred from the directions of fracture of rocks in testing machines, and also from geological faulting (Jaeger, 1956;





Price, 1973). It should be remembered that, because of the symmetry in  $\phi$  mentioned earlier, the theory leads to two possible planes of fracture equally inclined to the principal stresses and gives no reason for preferring either. For failure under combined stress in terms of the compressive and tensile strengths,  $C_0$  and  $T_0$  of the material, the relation

$$C_0/T_0 = (\sqrt{(u^2+1)} + u)/(\sqrt{(u^2+1)} - u)$$

holds. The theory predicts that the compressive strength of a material is always greater than its tensile strength but the ratio is rather smaller than that found in practice. The theory also predicts that under any conditions the normal to the plane of fracture makes the same angle  $\arctan(1/u)/2$ , with the direction of greatest principal stress. Although this is approximately true for compressive stresses, it is very far from the truth in the case of pure tension when the failure is usually by brittle fracture with the plane of fracture normal to the direction of tension. The reason may be that  $T_0$  should not be the actual tensile strength but the value at which shear failure in tension would take place if tensile fracture did not occur in practice before this value is reached. Nevertheless, the theory gives a reasonably accurate account of the behaviour of rocks under combined compressive stresses.

Mohr's representation of stress and failure is useful (Fig. 10). Mohr's theory assumes that at failure across a plane the normal and shear stresses across the plane,  $S_n$  and



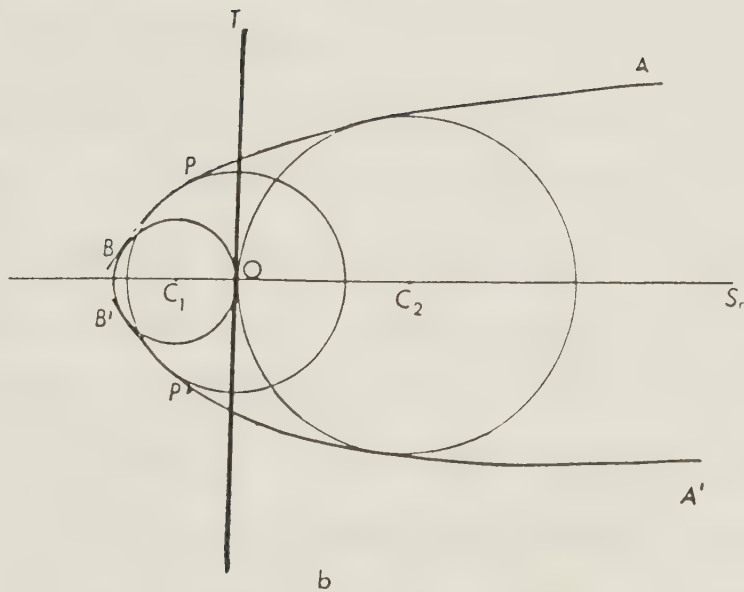
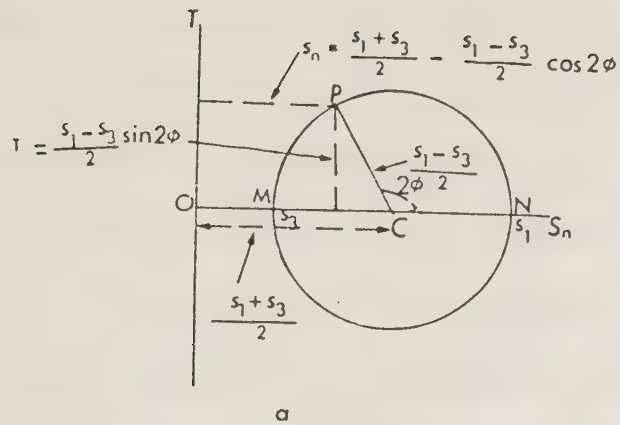


Figure 10.... Representation of stresses on any plane in a two-dimensional stress system by means of Mohr's stress circle.



$T$ , are connected by some relation  $T = f(S_n)$ . This relation may be plotted on a  $(S_n, T)$  plane and since changing the sign of  $T$  simply changes the direction of failure but not the condition for it, the curve is symmetrical about the  $S_n$ -axis. Any state of stress can be represented by a Mohr circle on the  $(S_n, T)$  plane. In representing the normal and shear stress across a plane in a biaxial stress field on the Mohr's Stress circle, the normal stress  $S_n$  and shear stress ( $T$ ) are chosen as co-ordinate axes. Here the greatest principal stress  $S_1$  and the least principal stress  $S_3$  are represented by  $ON$  and  $OM$  respectively on the  $S_n$ -axis. The quantity  $(S_1 + S_3)/2$  of Eqn. (2-6) represents the mid-point  $C$  on the  $S_n$  axis between  $M$  and  $N$ , while  $(S_1 - S_3)/2$  represents half the distance between  $M$  and  $N$ . If a circle is drawn with centre  $C$  and radius  $(S_1 - S_3)/2$ , then for any specific values of  $S_1$  and  $S_3$ , this circle represents the conditions of Equation 2-6 where  $2\phi$  is measured as indicated in Fig. 10 a. This construction as we shall see is used in representing the values of shear and normal stresses at failure. If this circle lies wholly within the failure envelope  $ABA'B'$  in Fig. 10 b, the stresses involved nowhere attain the critical values. If any portion lies outside it the material could not withstand the stresses. Obviously, the limiting case is that of a circle such as those of centres  $C_1$ ,  $O$  and  $C_2$  which just touch the curves  $AB$  and  $A'B'$ . In this case failure will take place under stress conditions corresponding to the points  $PP'$ , that is, over planes whose normals are inclined





at angles of half the angle PCN (Fig. 10a) to the direction of the greatest principal stress. The curve AB will be the envelope of all the circles corresponding to all conditions at which fracture takes place and for this reason is known as the *Mohr envelope*.

In principle, three circles which touch the envelope can be found from simple experiments. These are those of centres  $C_1$ , O and  $C_2$  (Fig. 10 b) , corresponding to tension, simple shear, and compression. But in practice it is difficult to perform shear or tensile tests on rock materials. An approximation to the Mohr envelope for many rocks, from results of triaxial tests, is the pair of straight lines

$$|T| = T_0 + u.S_n$$

corresponding to the Coulomb-Navier criterion for fracture under shear stress, where  $T_0$  is the shear strength. In this case the normal to the plane of fracture makes an angle (in the second quadrant)  $\arctan(1/u)/2$  with the direction of the greatest principal stress. When applied to triaxial stress, the Mohr theory leads to the result that only the Mohr circles for the plane containing the greatest and least principal stresses need be considered and that fracture always takes place in planes containing the direction of the intermediate principal stress. This is not altogether consistent with the experimental results.

One important application of these results is the study of faults which are fractures of the rocks of the earth's



crust (Fig. 11). Geologists distinguish three major types of faults and Anderson (1951) has shown that these are determined by the relative magnitudes of the principal stresses. One principal stress will always be vertical, and three cases arise according as this is the greatest, intermediate, or least principal stress. In the case where the vertical principal stress is the least in magnitude and the other two principal stresses are compressive (Fig. 11 A), the fault is known as a *thrust* fault. The planes of fracture pass through the direction of the intermediate principal stress and makes angles of less than  $45^\circ$  with the direction of the greatest compressive stress which is horizontal, (Fig. 11 B). This applies in regions of active tectonic compression or erosion or vertical unloading. When the intermediate principal stress is vertical, we have Figure 11 C. The failure can take place on either of two vertical planes (Fig. 11 D), which are equally inclined at angles of less than  $45^\circ$  to the direction of the greatest compressive stress. This type of fault is called a *transcurrent, wrench or strike-slip fault*. When the vertical principal stress is greatest as may often be the case at considerable depths, and in rift structures near the surface, the type of failure is called *normal faulting*. Here the faults make angles less than  $45^\circ$  with the vertical (Fig. 11 E).



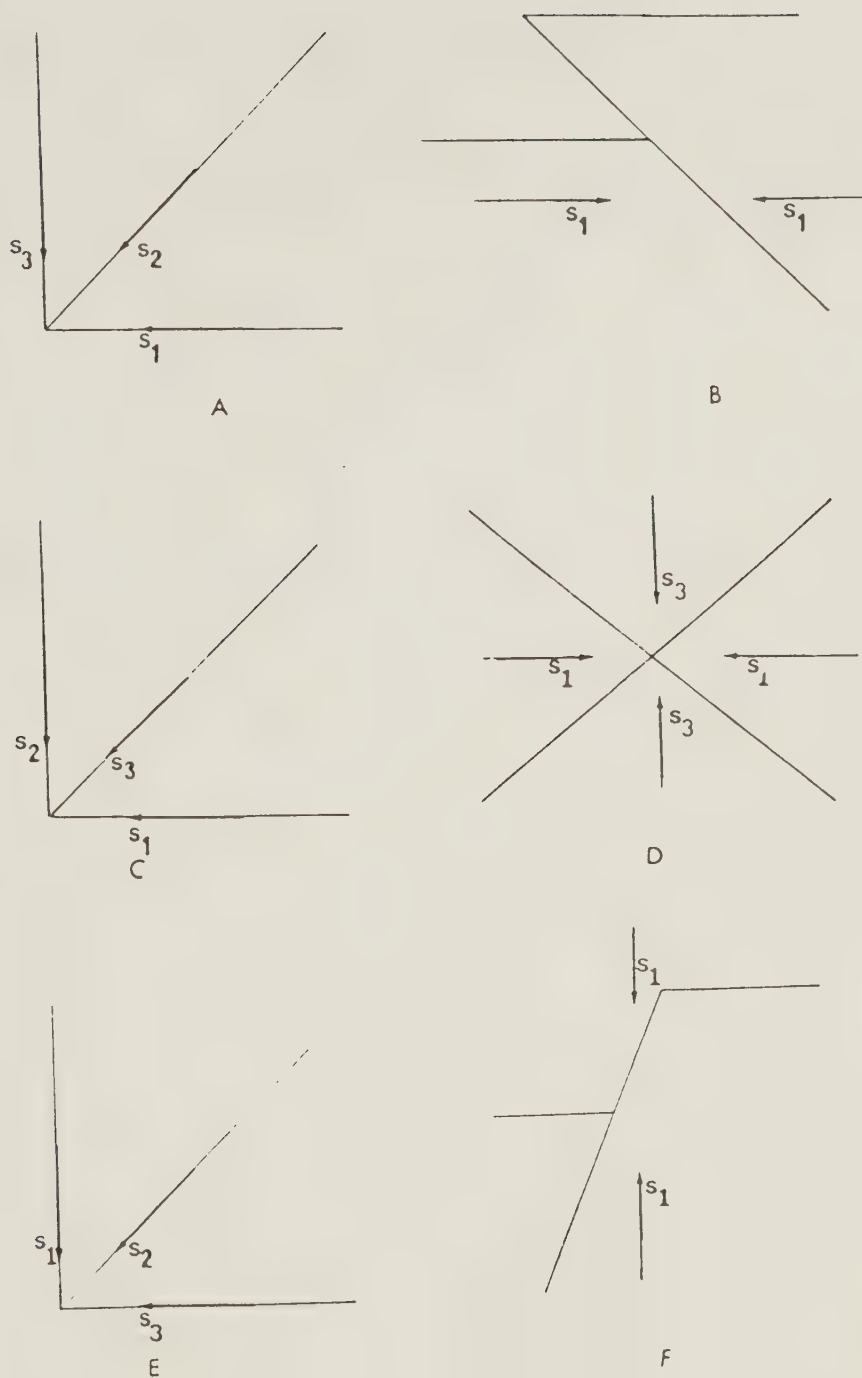


Figure 11.... Types of faulting in relation to the principal stresses; A and B, Thrust fault, C and D, Wrench fault, E and F, Normal fault.





## Stress Concentration Caused by the Borehole

The presence of a borehole distorts the pre-existing stress field, because as discontinuities in the earth's crust, boreholes cause redistribution of stresses in the nearby rock. An approximate calculation of this distortion is made by assuming that the rock is elastic, the borehole smooth and cylindrical and the borehole axis vertical and parallel to one of the regional principal stresses. Kirsch (1898) found an analytic solution of the problem of the stress field near a small hole of radius  $a$  in a large plate under uniaxial compression  $S$ . The solution as given by Timoshenko and Goodier (1951) is given below.

Let Figure 12 represent a plate subjected to a uniform compression of magnitude  $S$  in the  $x$ -direction. If a small circular hole is made in the middle of the plate, the stress distribution in the immediate vicinity of the hole will be changed. However, by Saint-Venant's principle we know that the change will be negligible at distances which are large compared with  $a$ , the radius of the hole. Let us consider the portion of the plate within a concentric circle of radius  $b$ , large in comparison with  $a$ . The stresses at the radius  $b$  are effectively the same as in the plate without the hole and are therefore given from Equations (2-2) and (2-3) by

$$(S_r)_{r=b} = S \cdot \cos^2 \phi = S/2 \cdot (1 + \cos 2\phi)$$

$$(T_{r\phi})_{r=b} = -1/2 S \cdot \sin 2\phi \text{-----2-9}$$

For stresses disposed symmetrically about a point of weakness in the earth's crust the direction of the principal



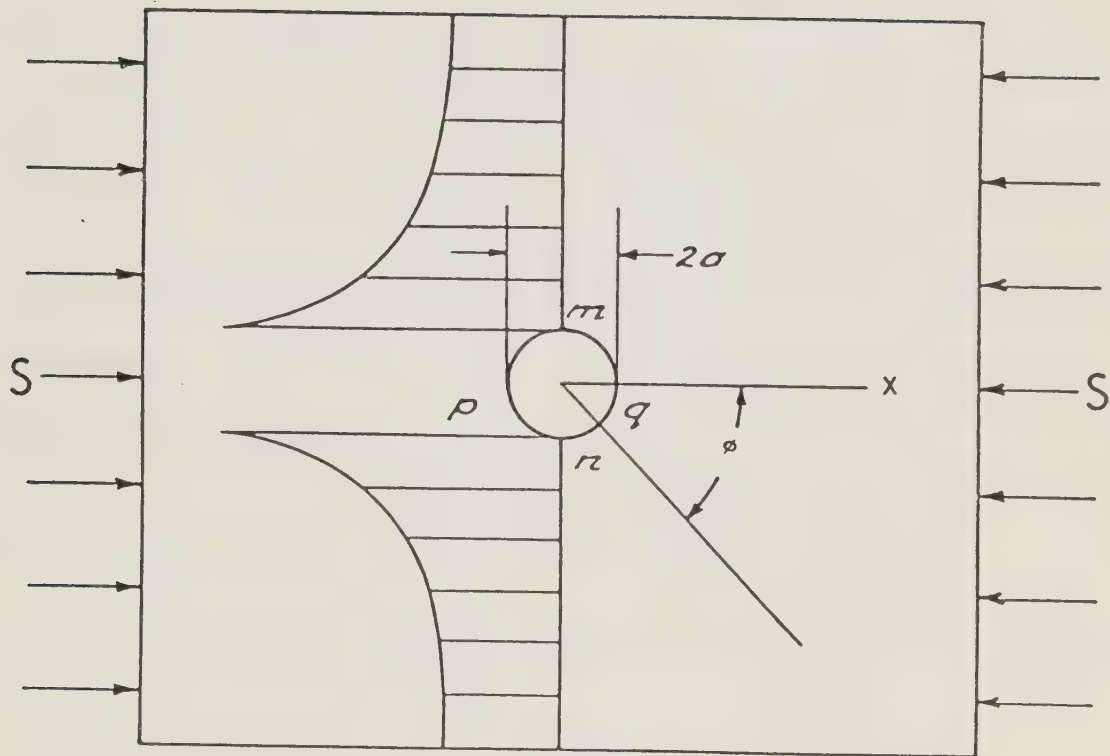


Figure 12.... A plate submitted to a uniform compression of magnitude  $S$  in the  $X$ -direction with a small hole in it



stresses will be vertical, radial, and tangential. The forces acting around the outside of the ring in Equation 2-9, having the inner and outer radii  $r=a$  and  $r=b$  give a stress distribution within the ring which can be considered as consisting of two parts. (a) The first is due to the constant  $1/2 S$  of the normal forces. (b) The remaining part consists of the normal forces  $1/2 S \cdot \cos 2\phi$  together with the shearing forces  $-1/2 S \cdot \sin 2\phi$ . The stresses produced by this second part may be derived from a stress function of the form

$$\Phi = f(r) \cdot \cos 2\phi \text{-----2-10}$$

This stress function can be substituted into the compatibility equation

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \left( \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} \right) = 0$$

to yield an ordinary differential equation from which  $f(r)$  can be determined.

$$(d^2/dr^2 + d/rdr - 4/r^2)(d^2f/dr^2 + df/rdr - 4f/r^2) = 0$$

The general solution is

$$f(r) = A \cdot r^2 + B \cdot r^4 + C/r^2 + D$$

Equation 2-10 then becomes

$$\Phi = (A \cdot r^2 + B \cdot r^4 + C/r^2 + D) \cdot \cos 2\phi \text{-----2-11}$$

The stress components are;

$$S_r = \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} = - \left( 2A + \frac{6C}{r^4} + \frac{4D}{r^2} \right) \cos 2\phi$$

$$S_\phi = \frac{\partial^2 \Phi}{\partial r^2} = \left( 2A + 12Br^2 + \frac{6C}{r^4} \right) \cos 2\phi \text{-----2-12}$$

$$T_{r\phi} = - \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \Phi}{\partial \phi} \right) = \left( 2A + 6Br^2 - \frac{6C}{r^4} - \frac{2D}{r^2} \right) \sin 2\phi$$





The constants of integration A,B,C,D are determined from the conditions of Eqn. 2-9 for the outer boundary and from the condition that the wall of the hole is free from external forces. These conditions give

$$2A + 6C/b^4 + 4D/b^2 = -S/2$$

$$2A + 6C/a^4 + 4D/a^2 = 0$$

$$2A + 6Bb^2 - 6C/b^4 - 2D/b^2 = S/2$$

$$2A + 6Ba^2 - 6C/a^4 - 2D/a^2 = 0$$

For an infinitely large plate  $a/b = 0$ . These four equations can then be solved to yield  $A=-S/4$ ,  $B=0$ ,  $C=-a^4.S/4$ ,  $D=a^2.S/2$ . These constants can now be substituted into equation 2-12, and by adding the stresses produced by the uniform compression  $S/2$  on the outer boundary we obtain

$$S_r = S/2 .(1-a^2/r^2) + S/2.(1+3a^4/r^4 - 4a^2/r^2).\cos 2\phi$$

as radial stress

$$S_\phi = S/2.(1+a^2/r^2)-S/2.(1+3.a^4/r^4).\cos 2\phi \text{ -----2-13}$$

as the tangential stress and

$$T_{r\phi} = -S/2.(1-3a^4/r^4+2a^2/r^2).\sin 2\phi$$

as the shearing stress, where the stresses are expressed in polar co-ordinates with the centre of the hole as the origin and the plane stress components are at a point  $(\phi, r)$  exterior to the hole of radius  $a$  in a plate under uniform uniaxial stress, with  $\phi$  measured from the axis of the compressive stress  $S$ .

If  $r \gg a$ ,  $S_r$  and  $T_{r\phi}$  approach the values given in equation 2-9. At the edge of the hole  $r=a$  we find

$$S_r = T_{r\phi} = 0 ; S_\phi = S-2S.\cos 2\phi \text{ -----2-14}$$



It can be seen from (2-14) that  $S_\phi$  is greatest when  $\phi = \pi/2$  or  $3\pi/2$  (i.e. when  $\cos 2\phi = -1$ ) that is at the ends m and n of the diameter perpendicular to the direction of the compression. At these points  $(S_\phi)_{\max} = 3S$  which is the maximum compressive stress and is three times the uniform stress  $S$  applied at the end of the plate. At the points p and q,  $\phi$  is equal to  $\pi/4$  and  $3\pi/4$  and we find  $S_\phi = -S$ . So there is a tensional stress, resulting from the applied compression, in the tangential direction at these points. For the cross section of the plate through the centre of the hole and perpendicular to the x-axis,  $\phi = \pi/2$  and from Equation 2-13

$$T_{r\phi} = 0 ; S_\phi = S/2.(2+a^2/r^2+3a^4/r^4) \text{ -----2-15}$$

It is clear that the effect of the hole is of a very localised character, and as  $r$  increases the stress approaches the value  $S$  very rapidly, within a few hole diameters. The distribution of this stress  $S_\phi$  as a function of  $r$  is shown in the Fig. 12 by the shaded area, regardless of the relative isotropy of a material. Having the solution equation 2-13 for compression or tension in one direction, the solution for compression or tension in two perpendicular directions can be easily obtained by superposition, of stresses given by (2-13) with  $(\phi+90)$  for the angular co-ordinate.

A closer approximation to the geological situation is shown in Fig. 13 where unequal compressions  $S$  and  $s$ ,  $S>s$ , are applied orthogonally to a plate with a small hole in it.



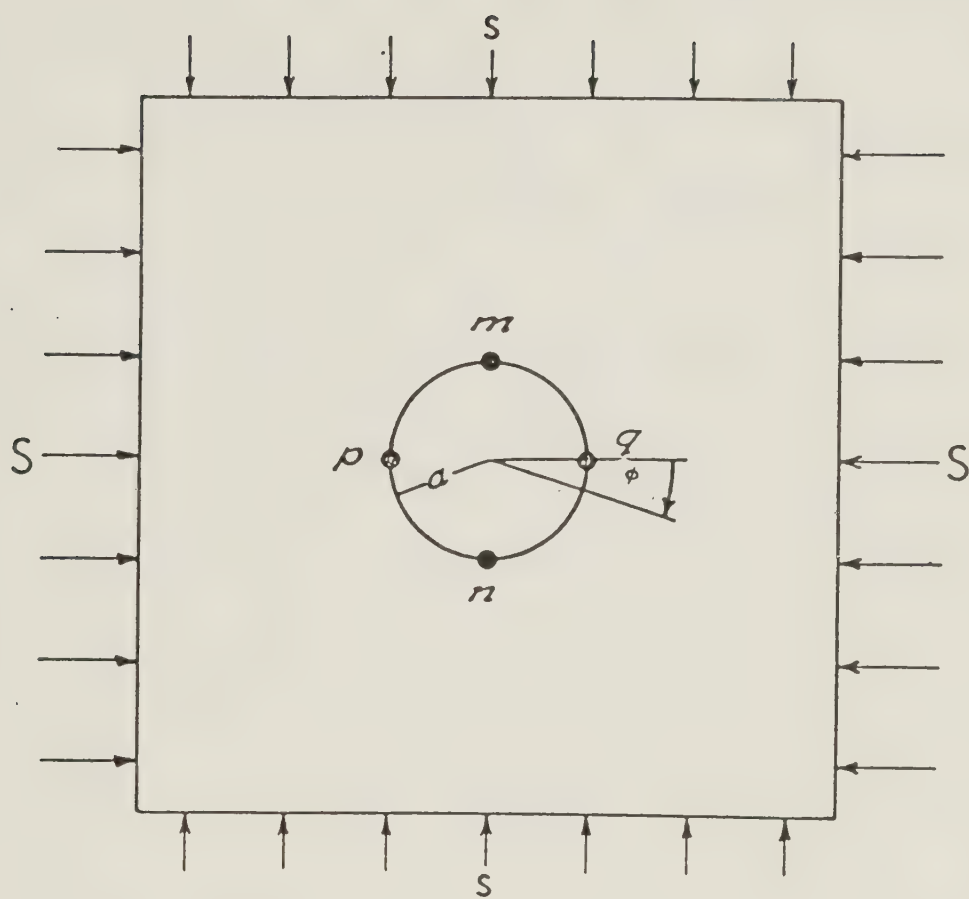


Figure 13.... A plate submitted to two compressions  $S$  and  $s$  with a small circular hole in it





At the hole boundary  $r=a$ , it follows from the superposition that

$S_r = T_{r\phi} = 0$  ;  $S_\phi = S+s-2(S-s)\cos 2\phi$ -----2-16  
 and  $S_\phi$  is maximum, when  $\cos 2\phi = -1$  with value  $(3S-s)$  when  $\phi = \pi/2$  ,  $3.\pi/2$  (at m, n), and minimum when  $\cos 2\phi = 1$ , with value  $(3s-S)$  when  $\phi = 0, \pi$  (at p, q).

Where measurements have been made, horizontal principal stresses in the upper crust range from equality  $S=s$  to about  $S=4s$  or more (McGarr and Gay, 1978). The values of the horizontal stresses across the principal planes in the neighbourhood of the borehole have been calculated by Gough and Bell (1982) for various relative values of the  $S/s$  ratio and are shown in Figures 14 and 15. Figures 14 A and B show the individual stress distributions. The principle of the superposition of the two parts of the stress field is illustrated in Fig. 14 C for the case in which  $S/s = 1.0$  . For  $S$  alone the tangential stress at the walls of the hole varies from a minimum value of  $-S$  (tensile) across the plane parallel to the  $S$ -axis to a maximum of  $+3S$ , across the plane normal to the  $S$ -axis. With the superposition, the stress field has radial symmetry and the tangential stress at the wall of the hole is  $+2S$  as can also be deduced from the equation 2-16. From equation 2-16 when  $S_\phi = 2S = 2s$ , it is independent of  $\phi$  and so failure may occur but without a preferred azimuth. As the ratio of  $S$  to  $s$  approaches one, the determination of the magnitudes and directions of  $S$  and  $s$  becomes less reliable because of the lack of resolution.



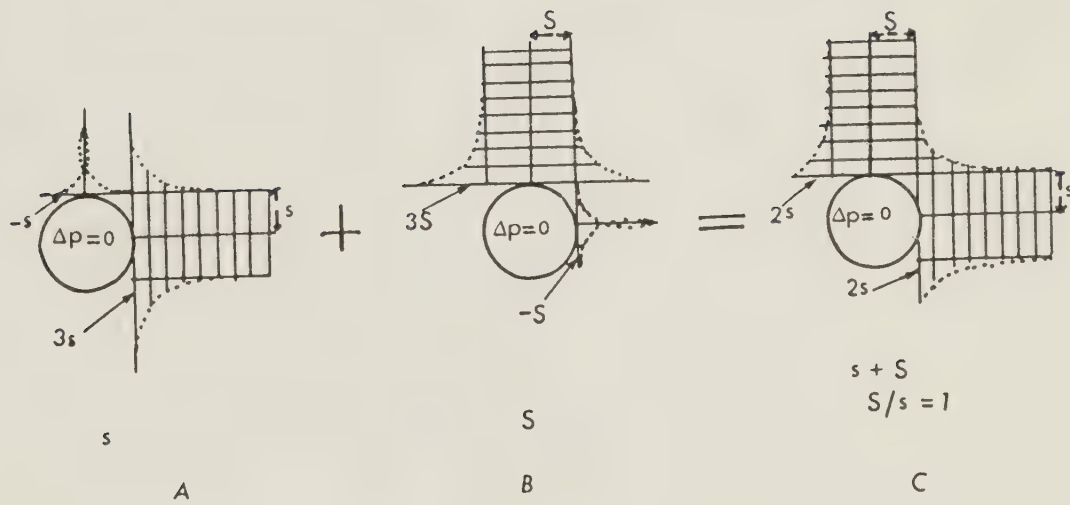


Figure 14.... Superposition of stress states about a borehole due to two horizontal principal stresses of equal magnitude.



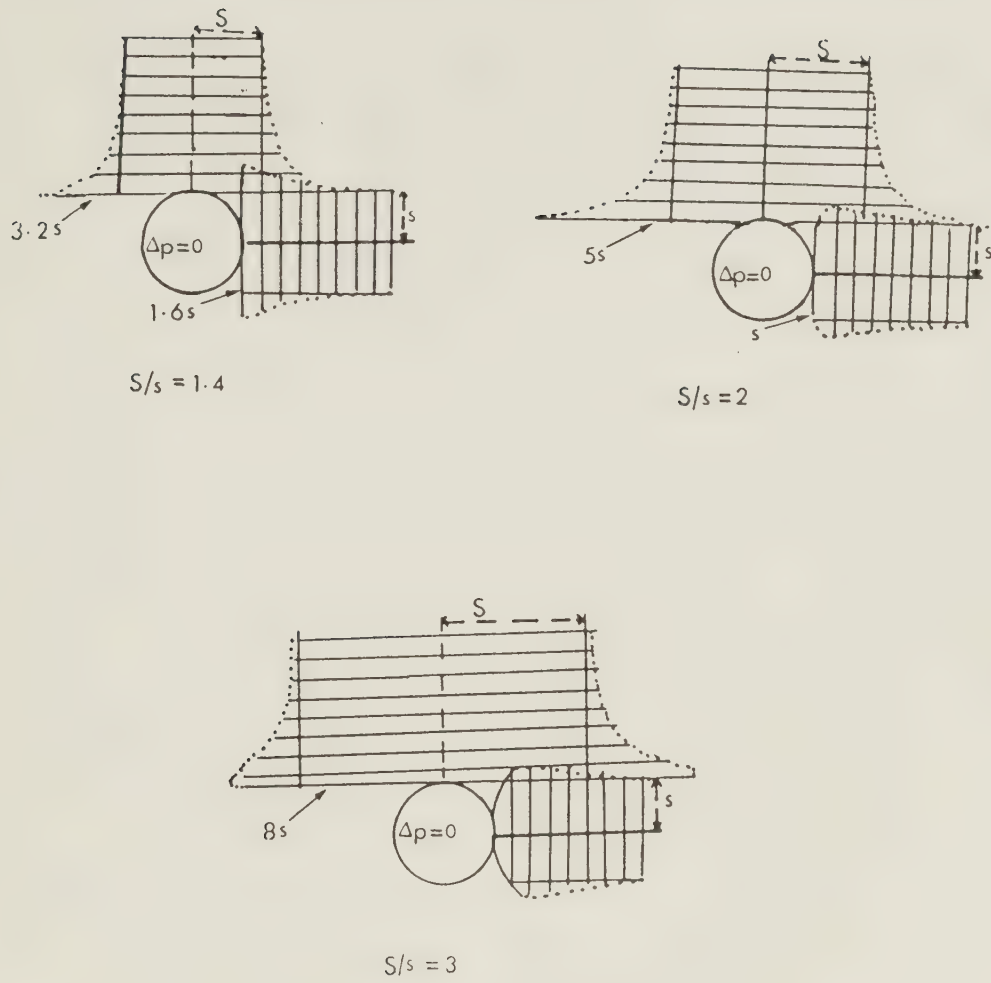


Figure 15.... Stress states about a borehole for regional stress-ratios  $S/s$  of 1.4, 2.0, and 3.0.





The resultant stress fields for other ratios of  $S/s$  are shown in figure 15. For the case when  $S/s = 3.0$  the tangential stress at the walls of the hole varies from a minimum of 0 to a maximum of  $+8s$  (see Eqn. 2-16).



### III. DATA COLLECTION AND ANALYSIS

Data have been secured from wells distributed widely in the sedimentary basin of Alberta as shown in Figure 16. In analysing the records it was observed that in depth ranges characterised by oversized holes, the tool tended to rotate in some parts and ceased to rotate in other parts. On the other hand, the tool ceased to rotate in certain ranges in which the two orthogonal diameters were equal indicating a circular cross-section. Such observations led to the adoption of three criteria to be used in identifying breakouts. These are:

1. The tool ceases to rotate, with the azimuth of the No. 1 electrode approximately constant.
2. The two orthogonal diameters are definitely unequal with the smaller diameter at bit size.
3. The tool was rotating both below and above the breakout zone. This ensures that the rotation was stopped by the breakout and not by some other cause.

The application of these three criteria causes the rejection of many elongated zones, but ensures that only definitely identified breakouts are used in the study. Table 3.1 lists the breakouts identified by means of these three criteria, from the available log records.

#### Representation and Statistics of Angular Data

We wish now to combine the various mean azimuths of the breakouts in order to obtain a representative breakout azimuth for each well. We may regard an angular observation



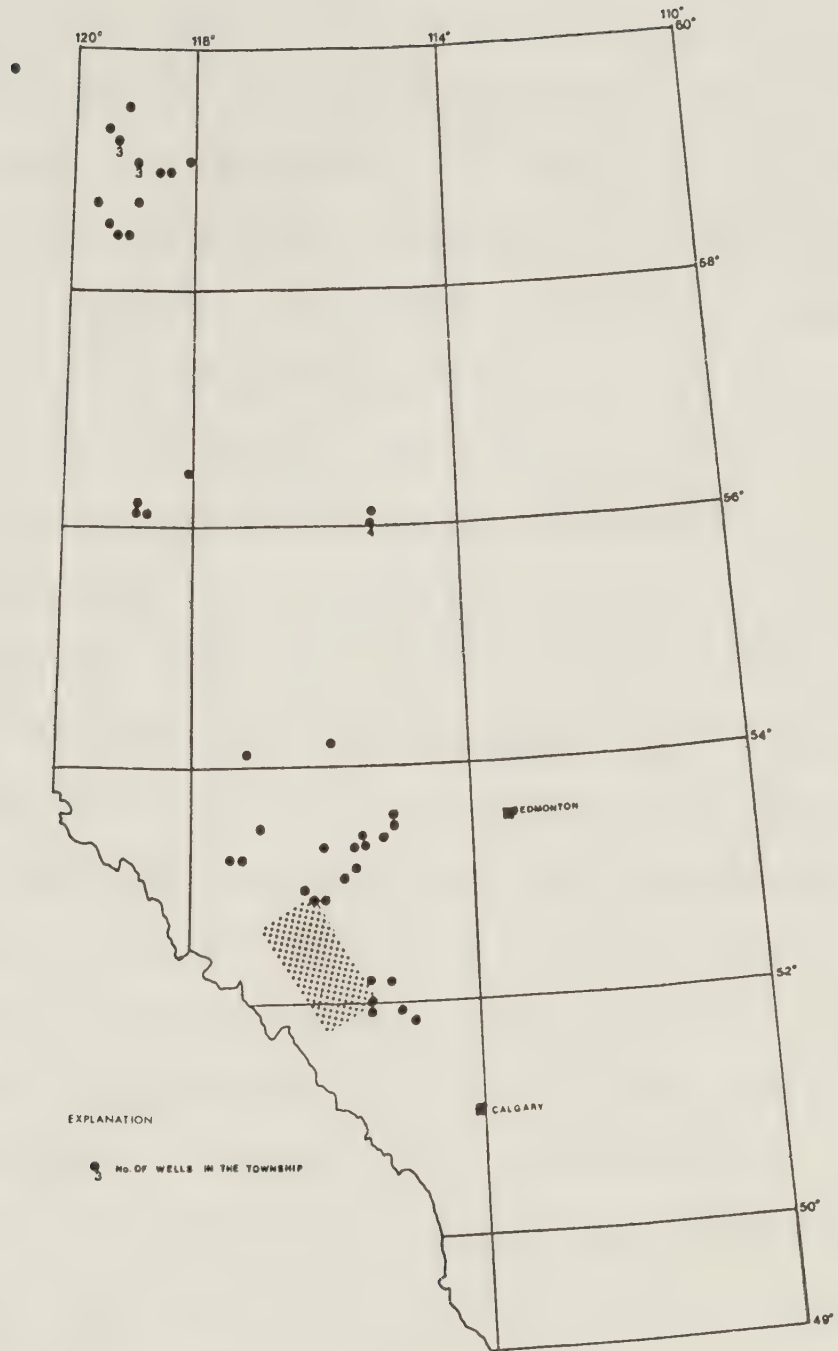


Figure 16.... Map of Alberta showing locations of wells analysed in this study





as a point on a circle of unit radius. A single observation  $\phi$  ( $0^\circ \leq \phi \leq 360^\circ$ ) measured in degrees is then a unit vector and the data can be described as circular data. If the vector is not directed, i.e. if the angles  $\phi$  ( $0^\circ \leq \phi \leq 180^\circ$ ) and  $180^\circ + \phi$  are not distinguished, the data can be described as *axial* or *non-polar* data. Ungrouped angular data can be represented in two ways; (a) by points on the circumference of a unit circle, the same weight being assigned to each observation, or (b) by drawing the radii of a unit circle obtained by joining the origin to the observed points on the circumference, (figure 17).

Alternatively, angular data can be grouped by dividing the range 0 to 360 degrees into a certain number of class intervals. The frequency is then the number of observations within each class. The data can then be represented on a histogram similar to that used on a line, by constructing a block whose area is proportional to the frequency in that interval on the circumference of a unit circle. This diagram is called a *circular histogram*. This histogram can also be unrolled so that it sits on a straight line divided into the class intervals. The point of cut for unrolling the unit circle needs careful selection. For data that have a mode (a preferred direction) it is reasonable to use a cut such that the centre of the linear histogram approximately corresponds to this mode to avoid division of the modal peak between the ends of the histogram. This linear histogram is preferred to the circular histogram mainly because it is easier to



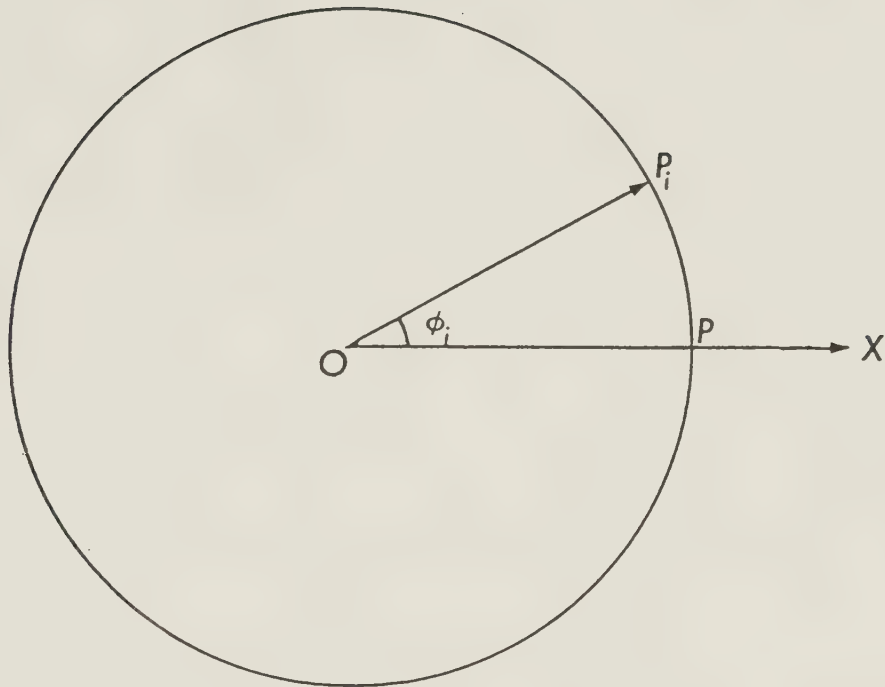


Figure 17.... Representations of the  $i$ th sample point  $\phi_i$   
 $OP=1$



evaluate.

Another natural representation of angular data is a Rose diagram. In this approach we construct a sector, corresponding to each class interval, with apex at the origin and radius proportional to the class frequency (and arc subtending the class interval). A disadvantage of this representation is that the area of each sector is proportional to  $N^2$ , where  $N$  is the frequency, so that the data appear much better grouped than they really are. However, the Rose diagram is widely used in geology.

#### A Measure of Location

In the first instance it is tempting to use the conventional measures on the line for a circular distribution. Let us consider a case where we have a sample of size 2 and the observed angles are  $1^\circ$  and  $359^\circ$ . The arithmetic mean and the sample variance give unreasonable results. Intuitively, however, we infer the mean direction and the deviation about the mean to be in some sense  $0^\circ$  and  $1^\circ$  respectively. A sensible answer, however, results if we select the zero direction as the y-axis in place of the x-axis. This will then reduce the data to  $269^\circ$  and  $271^\circ$ . Hence, the usual linear measures depend heavily on the choice of the zero direction and will not be appropriate for circular distributions. Let  $X_1, \dots, X_n$  be  $n$  observations on the line and let  $X_1', \dots, X_n'$  represent the same observations when the distances are measured from  $O'$  instead of the origin  $O$  along the x-axis. Let  $OO' = a$ . If  $L$  is a





suitable measure of location on the line we should have

$$L(X_1', \dots, X_n') = L(X_1, \dots, X_n) - \alpha \text{-----3-1}$$

which implies that the position of the point whose x-coordinate is  $L(X_1, \dots, X_n)$  remains invariant under the choice of origin. Similarly it is desirable that the measure of circular location should not depend upon the choice of the zero direction.

Let  $\phi_1', \dots, \phi_n'$  be the angles obtained from  $\phi_1, \dots, \phi_n$  with respect to a new zero direction OA. If angle XOA =  $\alpha$ , then we have

$$L(\phi_1', \dots, \phi_n') = (L(\phi_1, \dots, \phi_n) - \alpha) \bmod 2\pi \text{-----3-2}$$

The contrast between Equations 3-1 and 3-2 indicates the different natures of the problems on the line and on the circle.

### The Mean Direction

Suppose  $P_i$  is the point on the circumference of a unit circle corresponding to the angle  $\phi_i$ ,  $i = 1, \dots, n$ . Then the mean direction  $\bar{\phi}$  of  $\phi_1, \dots, \phi_n$  is defined as the direction of the resultant of the unit vectors  $\vec{OP}_1, \dots, \vec{OP}_n$ . The cartesian coordinates of  $P_i$  are  $(\cos\phi_i, \sin\phi_i)$  so that the centre of gravity of these points is  $(\bar{C}, \bar{S})$  such that

$$\bar{C} = 1/n \cdot \sum \cos\phi_i ; \bar{S} = 1/n \cdot \sum \sin\phi_i \text{-----3-3}$$

$$\text{If } \bar{R} = \sqrt{(\bar{C}^2 + \bar{S}^2)} \text{-----3-4}$$

then  $R = n\bar{R}$  is the length of the resultant and  $\bar{\phi}$  is the solution of the equations

$$\bar{C} = \bar{R} \cdot \cos\bar{\phi} ; \bar{S} = \bar{R} \cdot \sin\bar{\phi} \text{-----3-5}$$

It can be shown that  $\bar{\phi}$  has some desirable properties as a



measure of location.(see Appendix A)

### The Circular Variance

Let  $P_i$  be the point corresponding to  $\phi_i$  on a unit circle and  $\alpha$  be a fixed direction. If we assume initially that  $\alpha=0$  and suppose that  $P$  is the corresponding point on the circle then a measure of the circular dispersion between  $P$  and  $P_i$  is the smaller of the two angles that  $OP_i$  makes with  $OP$ , say  $\beta_i$  (see Fig. 17). We have

$$\beta_i = \min(\phi_i, 2\pi - \phi_i) = \pi - |\pi - \phi_i| \text{ -----3-6}$$

$1 - \cos\beta_i$  is an increasing function of  $\beta_i$ . Let

$$D = 1/n \cdot \sum (1 - \cos\beta_i) \text{ , } i = 1, \dots, n \text{ -----3-7}$$

be a measure of the dispersion of the points  $P_i$ . After shifting the zero direction to  $\alpha$  using

$$\phi_i' = (\phi_i - \alpha) \bmod 2\pi \text{ ,}$$

we can write

$$D = 1/n \cdot \sum (1 - \cos(\phi_i - \alpha)) \text{ -----3-8}$$

The dispersion  $D$  is minimised at  $\alpha = \bar{\phi}$ . If we equate the derivative of (3-8) with respect to  $\alpha$  to zero we have

$\sum \sin(\phi_i - \alpha) = 0$ , so that the dispersion is smallest about  $\bar{\phi}$  (see Appendix A Equations A-1 and A-6.). This is similar to the expression for the ordinary sample variance.  $D$  can be written about  $\bar{\phi}$  as  $S_0$  where

$$S_0 = 1 - 1/n \cdot \sum \cos(\phi_i - \bar{\phi}) \text{ -----3-9}$$

which can be simplified to

$$S_0 = 1 - \bar{R} \text{ -----3-10}$$

$S_0$  is called the circular variance and  $\bar{R}$  is the mean resultant vector magnitude. From (A-5), it can be seen that



$S_0$  is invariant under a change of the zero direction. Let  $R$  be the length of the resultant of the vectors  $OP_i$  i.e.  $R = n\bar{R}$  so that

$$S_0 = 1 - R/n \text{ -----3-11}$$

Thus we see immediately that

$$0 \leq S_0 \leq 1.0 \text{ -----3-12}$$

Since  $R$  tends to  $n$  for tightly grouped unit vectors, clearly  $S_0$  tends to 0 for tight distributions and  $S_0$  tends to 1.0 for random distributions. From (3-12)  $S_0$  takes values in  $(0,1)$  unlike  $\sigma^2$  for linear data whose range is  $(0,\text{infinity})$ . An appropriate transformation (first suggested by Von Mises in 1918) of  $S_0$  to the range  $(0,\text{infinity})$  is given by

$$\sigma = \sqrt{-2 \log_e (1 - S_0)} \text{ -----3-13}$$

In (3-13) it has been assumed that the range of  $\phi$  is  $(0, 2\pi)$ .

If the range of  $\phi$  is  $(0, 2\pi/L)$  then we define

$$\sigma = (-2 \log_e (1 - S_0))^{1/2} / L.$$

For grouped data we have

$$\bar{C} = 1/n \cdot \sum f_i \cdot \cos \phi_i ; \bar{S} = 1/n \cdot \sum f_i \cdot \sin \phi_i , \text{ and } \bar{R} = (\bar{C}^2 + \bar{S}^2)^{1/2}.$$

$$\text{Then } \cos \bar{\phi} = \bar{C} / \bar{R} ; \quad \sin \bar{\phi} = \bar{S} / \bar{R} , \quad S_0 = 1 - \bar{R} .$$

To obtain  $\bar{\phi}$  it is convenient to use

$$\bar{\phi} = \bar{\phi}' \text{ if } \bar{S} > 0 , \bar{C} > 0 \text{ (1st quadrant)}$$

$$\bar{\phi} = \bar{\phi}' + \pi \text{ if } \bar{C} < 0 , \bar{S} > 0 \text{ (2nd quadrant)}$$

$$\bar{\phi} = \bar{\phi}' + \pi \text{ if } \bar{C} < 0 , \bar{S} < 0 \text{ (3rd quadrant)}$$

$$\bar{\phi} = \bar{\phi}' + 2\pi \text{ if } \bar{S} < 0 , \bar{C} > 0 \text{ (4th quadrant)}$$

where  $\bar{\phi}' = \arctan(\bar{S} / \bar{C})$ , and  $-\pi/2 < \bar{\phi}' < \pi/2$ . For axial (non-polar) data, in the range  $(0^\circ - 180^\circ)$ , we first double



the angles to maintain the periodicity. Thus we have  $\phi_i' = 2\phi_i$  and carry out the calculations using the equations already listed, and the frequencies  $f_i$  corresponding to the mid-points  $\phi_i'$ . Suppose  $\bar{C}'$ ,  $\bar{S}'$ ,  $\bar{\phi}'$  etc. are the quantities for  $\phi_i'$  corresponding to  $\bar{C}$ ,  $\bar{S}$ ,  $\bar{\phi}$  etc. then an appropriate measure of the circular mean  $\bar{\phi}$  for the  $\phi_i$  can be taken as  $\bar{\phi} = \bar{\phi}'/2$  and an appropriate measure of the circular variance  $S_0$  for  $\phi_i$  is  $S_0 = 1 - (1 - S_0')^{1/4}$ . In general if the range of  $\phi$  is  $(0, 2\pi/L)$  we have

$$S_0 = 1 - (1 - S_0')^{1/2}.$$

The result of applying these statistics to the raw azimuth data measured at every two feet depth interval within an identified breakout is shown in the column headed MEAN in Table 3.1 and also in columns 8 and 9 of Table 3.2. Table 3.2 shows the mean orientation of the various breakout zones in a well.

### Statistical Decision

We are now near a position to make inferences on our data. Before making any inferences, however, we wish to look at the problem of the *regression* of one variable, in our case the azimuth, on another variable (independent variable), which is the depth in our situation. We shall then look at the correlation or the degree of relationship between the variables. We first consider the problem of how well a straight line explains the relationship between two variables. This requires the equation for the Least Square Regression Line. The least square regression line of Y on X





TABLE 3.1

Breakout azimuths in relation to rock types and age of rocks. (All lengths are in feet except where indicated).

W attached to a breakout depth range indicates that a section of the range shown does not satisfy the three criteria. N is number of observations.

WELL 6-30-46-17W5				80 ELONGATIONS				51 BREAKOUTS				STRATIGRAPHIC INFORMATION				FORMATION	LITHOLOGY	
ELONGATED INTERVALS				BREAKOUT INTERVALS				BREAKOUT INTERVALS				PERIOD	Fm. TOP					
DEPTH RANGE	LENGTH	DEPTH RANGE	LENGTH	DEPTH RANGE	LENGTH	N	MEAN AZIMUTH(DEC)	DEPTH RANGE	LENGTH	N	MEAN AZIMUTH(DEC)							
3229-3541	312	3254-3521e	174	3254-3521e	174	28	138.0											
3719-3750	31	3722-3732	10	3722-3732	10	3	134.3											
3754-3792	38	3760-3790	30	3760-3790	30	6	51.2											
3816-4069	253	3828-4050e	178	3828-4050e	178	32	137.3											
4076-4145	69	4106-4120	14	4106-4120	14	4	74.5											
4186-4526	340	4212-4488e	244	4212-4488e	244	38	11.6											
4564-4646	82	4584-4630e	22	4584-4630e	22	10	141.4											
4659-4728	69	4662-4724	62	4662-4724	62	9	149.9											
4730-4762	32	30-40.56-62	16	30-40.56-62	16	5	153.0											
4861-5144	283	4862-5132e	174	4862-5132e	174	30	145.9											
5147-5176	29	5150-5168	18	5150-5168	18	3	139.7											
5228-5326	108	5246-5332e	24	5246-5332e	24	10	142.7											
5340-5370	30	5352-5368	16	5352-5368	16	5	39.2											
5526-5820	294	5546-5812e	160	5546-5812e	160	40	144.9											
5852-5935	83	5852-5922	12	5852-5922	12	3	34.3											
5982-6002	20	5984-5996	14	5984-5996	14	7	154.1											
6004-6053	49	6004-6028	24	6004-6028	24	7	154.1											
6058-6075	17	6058-6072	84	6058-6072	84	4	35.0											
6100-6340	240	6156-6320e	84	6156-6320e	84	26	162.6											
6374-6452	78	6394-6438e	28	6394-6438e	28	8	76.0											
6504-6526	22	6514-6522	8	6514-6522	8	3	150.3											
6546-6576	30	6548-6570	22	6548-6570	22	4	75.5											
6607-6781	174	6610-6744e	82	6610-6744e	82	18	140.5											
6785-6804	19	6786-6796	10	6786-6796	10	3	152.3											
6814-7076	262	6920-7028e	46	6920-7028e	46	12	157.8											
7118-7192	74	7230-7284	24	7230-7284	24	6	158.5											
7208-7243	35	7218-7230	14	7218-7230	14	3	135.7											
7350-7448	88	68-80.7412-26	26	68-80.7412-26	26	8	139.8											
7450-7558	108	7484-7504	10	7484-7504	10	5	142.0											
7615-8006	391	7710-7996e	152	7710-7996e	152	36	135.4											
8034-8385	351	8036-8378e	280	8036-8378e	280	49	151.7											
8426-8640	414	8430-8586e	400	8430-8586e	400	49	154.1											
8648-8874	26	8858-8866	12	8858-8866	12	3	78.3											
8876-9210	334	8978-9124e	128	8978-9124e	128	20	134.3											
9226-9284	58	9244-9274	30	9244-9274	30	4	141.5											
9346-9354	8	9346-9350	4	9346-9350	4	2	142.0											
9430-9436	6	9430-9436	6	9430-9436	6	3	141.0											
9491-9553	62	9498-9512	14	9498-9512	14	3	154.7											
9576-9645	69	32-38.40-44	10	32-38.40-44	10	5	91.2											
9662-9672	9	9668-9672	6	9668-9672	6	3	129.0											
9701-9895	294	9718-9976e	128	9718-9976e	128	27	131.6											
1006-10319	313	10006-10252e	68	10006-10252e	68	15	133.4											
10359-10759	400	10368-10758e	164	10368-10758e	164	29	118.9											
10800-10844	44	10804-10808	4	10804-10808	4	2	169.0											
11000-11050	50	11008-11020	12	11008-11020	12	5	28.2											
11411-11424	13	11414-11418	4	11414-11418	4	2	141.0											
11810-11826	16	11812-11820	8	11812-11820	8	4	144.5											
11830-11848	18	11834-11842	8	11834-11842	8	4	123.5											
11886-11905	8	11888-11904	6	11888-11904	6	3	148.7											
11926-11956	30	11944-11948	4	11944-11948	4	2	146.0											
11970-11980	10	11970-11976	6	11970-11976	6	3	129.3											
LONGATION ENDS 12010, LO FERNIE, TOTAL DEPTH 15820 CAMBRIAN PROD. ZONE LD BLAIR.																		

ELONGATION ENDS 12010. LO FENIE. TOTAL DEPTH 15820 CAMBRIAN PROD. ZONE LD BLAIR.



WELL 7-9-49-24WS			144 ELONGATIONS			71 BREAKOUTS			STRATIGRAPHIC INFORMATION		
ELONGATED INTERVALS			BREAKOUT INTERVALS								
1481-1503	22	1487-1497	10	5	146.0						
1511-1535	24	1531-1535	4	3	146.3						
1713-1921	208	1719-1813e	114	60	89.3						
1863-2018	55	1867-2011e	28	14	135.6						
2097-2153	56	2101-15-23-27	20	10	135.7						
2152-2221	67	2152-2221	36	18	82.9						
2257-2295	38	2257-2273	16	9	91.0						
2315-2327	12	2317-2327	10	6	143.6						
2351-2457	106	2383-2451e	52	27	136.3						
2551-2587	36	2567-2575	8	5	73.0						
2606-2683	77	2667-2681	14	8	60.0						
2759-2820	61	2749-2809	34	12	114.3						
2970-3568	518	2979-3527e	366	189	170.2						
3590-3643	53	3581-3599	8	5	67.2						
3681-3963	282	3683-5107e	268	45	29.5						
3967-3973	6	3967-3973	6	4	39.0						
4008-4087	78	4055-4068	14	8	119.1						
4198-4222	24	4201-4217	18	10	125.0						
4228-4276	48	4228-4276	26	15	132.7						
4277-4334	57	4277-4334	36	20	38.9						
4346-4427	81	4377-4417	40	21	22.8						
4450-5107	657	4509-5003e	62	38	162.2						
5300-5880	590	5309-5875e	96	55	79.1						
5928-7311	1383	5983-7309e	248	128	136.8						
7312-7676	364	7313-7667e	60	30	134.1						
7682-7704	22	7683-7697	14	7	123.1						
7710-7840	130	7717-7811e	22	11	130.5						
7857-7867	110	7861-7883	22	11	146.6						
7884-8046	62	7989-7997	8	4	49.8						
8102-8124	22	8107-8115	6	3	88.0						
8202-8221	19	8207-8215	6	3	144.6						
8242-8255	13	8245-8251	6	3	134.0						
8300-8316	16	8301-8311	10	5	142.2						
8360-8498	138	8445-8451	6	3	146.1						
8640-8498	0	8459-8478	20	10	146.1						
8740-8792	52	8740-8792	12	6	168.0						
9469-9497	28	9471-9477	6	3	127.3						
9545-9555	10	9549-9555	6	3	96.3						
10321-10358	37	10323-10339	6	3	112.0						
10376-10415	39	10383-10387	4	2	2.0						
10478-10493	71	10491-10543	58	23	160.6						
10563-10685	122	10563-10580	66	34	53.0						
10687-10857	169	10705-10825e	26	45	159.8						
10860-10893	33	10863-10867	4	2	133.5						
10916-10923	7	10919-10923	4	2	155.0						
10999-11133	134	11021-11071	50	25	125.6						
11143-11167	24	11147-11155	8	4	154.3						
11173-11195	22	11177-11185	8	4	141.0						
11217-11252	35	11219-11223	8	4	154.7						
11442-11527	85	11457-11501e	24	12	177.2						
11545-11570	25	11553-11568	16	9	35.4						
11573-11703	130	11657-11685	26	14	33.4						
11720-11798	79	11737-11791e	56	29	180.0						
11801-11858	57	11801-11833e	22	11	156.3						
12035-12065	30	12047-12093	12	6	146.5						
12129-12150	21	12133-12141	8	4	142.6						
12270-12295	25	12291-12297	6	3	134.3						
12387-12403	16	12397-12401	4	2	169.0						
12410-12419	9	12413-12417	4	2	147.5						
12650-12683	33	12665-12675	10	5	45.6						
12713-12798	85	12723-12735	12	6	174.0						
12813-12953	140	12761-12765	4	2	96.5						
12965-13000	35	12965-12993	18	9	154.8						
13022-13040	18	13025-13029	4	2	174.0						
13276-13281	5	13277-13281	4	2	44.0						
13480-13496	16	13491-13495	4	2	163.7						
13502-13512	10	13505-13511	6	3	152.7						
13537-13554	17	13548-13553	4	2	137.0						
13822-13851	29	13837-13845	8	4	136.5						
13856-13887	31	13871-13873	2	1	152.0						
ELONGATION ENDS 13887, NIKANASIN			TOTAL DEPTH *****			NIKANASIN; D84					



WELL 12-19-53-9WS				11 ELONGATIONS				10 BREAKOUTS				STRATIGRAPHIC INFORMATION			
ELONGATED INTERVALS		BREAKOUT		INTERVALS											
6489-6550	61	6508-6540	32	16	117.9										
6606-6636	30	6608-6636e	20	10	121.3										
6706-6736	30	6712-6736	24	10	119.7										
6746-6780	34	6746-6774	28	14	119.4										
6854-6882	28	6854-6882	28	14	124.4										
6910-6980	70	6910-6980	70	36	127.6										
7226-7280	54	7226-7280	54	36	124.9										
7410-7456	46	7412-7456	46	23	125.7										
7513-7530	17	7516-7524	17	8	137.5										
7782-7800	18	7792-7796	18	4	145.0										
WELL B-45-A-94-P-14															
13 ELONGATIONS				6 BREAKOUTS											
5726-5729	3	5727-5729	2	1	149.0	U. DEV.	4328					FT. SIMP			
5845-5858	13	5845-5851	6	5	158.3	U. DEV.						FT. SIMP			
5883-5895	12	5885-5893	8	5	68.5	U. DEV.						FT. SIMP			
5913-5923	10	5913-5921	8	4	63.8	U. DEV.						FT. SIMP			
5930-5938	8	5931-5937	6	3	53.7	U. DEV.						FT. SIMP			
5954-5956	4	5955-5957	2	1	61.0	U. DEV.						FT. SIMP			
ELONGATION ENDS 5958, FT. SIMP. TOTAL DEPTH 6089, SLAVE POINT-6050 PROD. ZONE, SLAVE POINT (L DEV.)															
WELL 6-12-50-11WS															
16 ELONGATIONS				12 BREAKOUTS											
2480-2497	17	81-87-89-97	14	13	120.4	MISSI/U. DEV	2318/2481					BANFF/WABAMUN			
2497-2505	8	2497-2505	8	5	129.3	U. DEV.						WABAMUN			
2507-2516	9	2507-2512	5	5	125.6	U. DEV.						WABAMUN			
2526-2535	9	2528-2535e	4	4	124.3	U. DEV.						WABAMUN			
2535-2552	47	2536-2577e	24	24	121.7	U. DEV.						WABAMUN			
2585-2588	3	2585-2588	3	2	133.0	U. DEV.						WABAMUN			
2596-2611	15	2608-2610	2	2	123.0	U. DEV.						WABAMUN			
2615-2627	12	2615-2619	4	4	126.5	U. DEV.						WABAMUN			
2676-2680	5	2676-2678	3	3	132.0	U. DEV.	2699					WABAMUN			
2702-2720	18	2702-2720	18	9	137.6	U. DEV.						BLUERIDGE			
2721-2731	10	2722-2730	8	8	137.6	U. DEV.						BLUERIDGE			
2731-2735	4	2732-2735	3	3	136.0	U. DEV.						BLUERIDGE			
ELONGATION ENDS 2874, IRETON TOTAL DEPTH 2876, IRETON=2858 PROD. ZONE, NISKU =2736m (U. DEV)															
WELL 14-20-50-12WS															
28 ELONGATIONS				24 BREAKOUTS											
7425-7429	4	7425-7429	4	2	129.5	L. CRETACEOUS	7030					BLAIR			
7433-7439	6	7433-7435	2	2	123.0	L. CRE.						BLAIR			
7465-7547	82	7477-7548e	54	26	126.2	L. CRE.	/7527					BLAIR/GLAUC			
7577-7596	19	7584-7590	6	3	124.7	L. CRE.	7569					L MANV			
7651-7709	58	7652-7704	52	23	122.9	JURASSIC	7626/7677					BLS Q/U FERNIE			
7717-7737	20	7717-7737	14	14	127.3	JURA.	7707					ROCK CK SS.			
7751-7783	32	7752-7780	28	10	125.2	JURA.						SS.			
7827-7858	41	7828-7848-84	24	18	120.0	JURA.						SS.			
8116-8156	8	8150-8156	6	3	120.3	MISSISSIPP.	7828					SS.			
8161-8197	16	8188-8196	8	6	106.5	MISS	8150					SS.			
8215-8237	22	8218-8228	10	5	106.6	MISS						SS.			
8255-8272	17	8256-8268	12	5	106.6	MISS						SS.			
8280-8295	15	8286-8290	4	2	119.0	MISS						SS.			
8445-8454	9	8446-8452	3	3	120.0	MISS						SS.			
8678-8684	170	8696-8814e	56	3	120.0	MISS						SS.			
8851-8864	13	8862-8862	10	5	124.7	U. DEV.	8680					SS.			
8895-8913	18	8896-8910	14	7	123.7	U. DEV.						SS.			
8937-8945	8	8938-8942	4	2	118.0	U. DEV.						SS.			
8946-8963	34	8968-8974	6	3	124.3	U. DEV.						SS.			
9097-9136	41	9100-9138e	30	15	119.3	U. DEV.						SS.			
9207-9227	20	9208-9214	6	3	120.5	U. DEV.						SS.			
9237-9251	14	9238-9242	4	2	131.0	U. DEV.						SS.			
9367-9473	86	9382-9464e	48	24	117.0	U. DEV.	9388					SS.			
9487-9501	14	9488-9496	10	5	124.4	U. DEV.						SS.			
ELONGATION ENDS 9501, BLUERIDGE TOTAL DEPTH 11537, CAMBRIAN=11483 PROD. ZONE NISKU=9532 / L MANV=7569 (U. DEV/L. CRE)															
WELL 4-26-51-11WS															
2 ELONGATIONS				2 BREAKOUTS											
8585-8607	22	8585-8605	20	11	128.1	U. DEVONIAN	8523					WINTERBURN/GRAMINIA			
8631-8665	34	8631-8663	32	17	114.6	U. DEV.	8652					CALMAR			
ELONGATION ENDS 8665, CALMAR TOTAL DEPTH 920 IRETON=9062 PROD. ZONE NISKU=8662															





WELL 3-32-47-13WS										26 ELONGATIONS										23 BREAKOUTS										STRATIGRAPHIC INFORMATION																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																				
ELONGATED INTERVALS					BREAKOUT INTERVALS					ELONGATIONS					BREAKOUTS					ELONGATIONS					BREAKOUTS					ELONGATIONS					BREAKOUTS					STRATIGRAPHIC INFORMATION																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																										
9579-9599	20	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9587	8	9579-9



WELL 10-27-45-18WS			20 ELONGATIONS			14 BREAKOUTS			STRATIGRAPHIC INFORMATION		
ELONGATION INTERVALS			BREAKOUT INTERVALS								
12149-12503	354	12233-12495e	106	57	143.1	MISSISSIPPI	12109	PEKISKO			
12527-12665	138	12565-12611	46	24	150.2	MISS.	12270	BANFF			
12759-13500	741	12789-13499e	314	157	137.0	/MISS./U.DEV.	/12777/12787	BANFF/EXSHAW/WABAMUN			
13501-13865	364	13501-1361e	324	161	131.0	U. DEVONIAN	13502	NISKU			
13865-14103	218	13865-14095	210	106	132.8	U. DEV.	13502/14008	NISKU/IRETON			
14129-14151	22	14131-14145	14	7	130.4	U. DEV.	14008	IRETON			
14153-14227	94	14153-14247	94	47	132.4	U. DEV.		IRETON			
14263-14285	32	14275-14285	20	10	132.4	U. DEV.		IRETON			
14301-14341	39	14313-14338	26	12	135.0	U. DEV.		IRETON			
14399-14431	32	14399-14423	24	14	131.7	U. DEV.		IRETON			
14659-14669	10	14659-14669	10	5	143.0	U. DEV.	14659	DUVERNAY			
14729-14841	112	14737-14835e	60	33	133.2	U. DEV.	14659/14788	DUV./B.H.L.			
14859-14901	42	14861-14869	8	4	143.0	U. DEV.	14788	BEAVERHILL LAKE			
15101-15407	306	15107-15399e	86	44	138.8	U. DEV.	/15162	B.H.L./SWAN HILLS			
ELONGATION ENDS 15407, CAMBRIAN; TOTAL DEPTH 15448, CAMBRIAN=15245						D & A.					
WELL 13-9-52-8WS			36 ELONGATIONS			20 BREAKOUTS					
1526-1594	68	1540-1584	24	14	139.2						
1612-1620	8	1612-1620	8	3	144.7						
1628-2284	658	1636-2294e	422	104	145.9						
2358-2435	37	2388-2424e	6	3	128.7						
2440-2462	22	2456-2462	30	11	132.5						
2474-2550	76	2476-2532e	12	6	132.8						
2638-2670	32	2654-2666	122	26	128.2						
2754-2900	146	2770-2900e	12	6	128.2						
2906-2942	36	2908-2920	48	22	91.1						
2950-3140	190	2986-3094e	23	7	45.1						
3142-3200	58	3142-3200	70	24	175.7	U. CRETACEOUS 3413		LEA PARK			
3294-3496	202	3412-3492e	16	8	134.1	U. CRF.	3413/3818/4317	LEA PARK			
3589-3641	72	3616-3632	454	110	135.5	U. CRF.	6762	LEA/1st WS/CARD ZONE			
3700-4376	676	3730-4342e	90	24	130.8	L. CRE.		D-1			
6782-6864	102	6764-6864e	12	5	125.0	L. CRE.		D-1			
6952-6976	32	6960-6972	12	3	120.7	L. CRE.		D-1			
6980-7012	27	6988-6994	12	4	128.8	L. CRE.		D-1			
7015-7042	50	7030-7042	36	13	124.6	L. CRE.		D-1			
7082-7132	58	7086-7130	40	10	125.5	L. CRE.		D-1			
7136-7194	58	7154-7194	40	10	125.5	L. CRE.		D-1			
ELONGATION ENDS 7194, D-1; TOTAL DEPTH 8100, IRETON=7999; (U. DEV) PROD. ZONE NISKU = 7854 (U. DEV)											
WELL 8-8N-51-9WS m			11 ELONGATIONS			8 BREAKOUTS					
2247-2255	13	2247-2249	2	2	145.5	U. DEVONIAN	2212.3m	WABAMUN			
2256-2258	3	2256-2258	2	2	139.0	U. DEV.		WABAMUN			
2287-2290	3	2287-2289	2	2	131.0	U. DEV.		WABAMUN			
2296-2302	7	2296-2300	4	4	127.0	U. DEV.		WABAMUN			
2343-2350	7	2344-2348	5	5	128.0	U. DEV.		WABAMUN			
2431-2440	9	2431-2438	8	8	140.0	U. DEV.	2419.5m	BLUERIDGE			
2442-2445	3	2442-2444	2	2	118.0	U. DEV.		BLUERIDGE			
2446-2452	6	2446-2451	5	5	140.6	U. DEV.		BLUERIDGE			
ELONGATION ENDS 2451.5, BLUERIDGE; TOTAL DEPTH 2654m IRETON=2584m											
WELL 6-20-50-15WS			46 ELONGATIONS			41 BREAKOUTS					
9149-9161	12	9151-9159	6	4	143.0	JURA/MISS.	8972/9157	NORDEGG/SHUNDA			
9167-9171	4	9168-9171	2	1	134.5	MISSISSIPPI	9157	SHUNDA			
9172-9181	8	9173-9181	2	5	143.0	MISS.		SHUNDA			
9195-9199	4	9195-9198	4	3	140.3	MISS.		SHUNDA			
9201-9205	4	9201-9205	4	3	137.0	MISS.		SHUNDA			
9229-9233	4	9228-9233	16	9	123.3	MISS.		SHUNDA			
9237-9253	16	9237-9253	24	12	120.9	MISS.		SHUNDA			
9267-9291	24	9267-9291	64	43	124.0	MISS.		SHUNDA			
9297-9361	64	9297-9361	2	1	9157/9330	MISS.		SHUNDA/PEKISKO			
9411-9417	6	9411-9413	2	1	9330	MISS.		PEKISKO			















STRATIGRAPHIC INFORMATION

WELL 7-18RN-109-7W6 12 ELONGATIONS 9 BREAKOUTS

ELONGATED INTERVALS	BREAKOUT INTERVALS	
4912-4922	4912-4920	10 5 15 5
5040-5074	5048-5072	34 30 15 172.0
5516-5562	5522-5552e	46 40 20 16.0
5604-5614	5604-5612	10 5 12.6
5628-5638	5628-5636	10 5 11.2
5740-5750	5740-5748	10 5 27.2
5780-5790	5780-5788	10 5 13.4
5800-5810	5800-5808	10 5 25.7
5850-5886	5864-5872	36 10 5 35.4

WELL 10-17RN-110-9W6 11 ELONGATIONS 5 BREAKOUTS

4140-4288	4142-4280	148 140 70 41.7
4290-4358	4290-4350	68 60 30 58.0
4368-4410	4374-4402	42 30 15 88.0
4452-4564	4468-4564	112 100 50 56.0
5656-5700	5656-5684	44 30 15 164.0

WELL 2-20-81-9W5 7 ELONGATIONS 5 BREAKOUTS

4084-4210	4084-4192	126 110 55 117.8
4224-4264	4224-4262	40 40 20 118.9
4268-4364	4268-4376	118 110 55 114.6
4414-4564	4494-4552	150 60 30 111.3

WELL 4-15-81-9W5 7 ELONGATIONS 3 BREAKOUTS

4908-4982	4912-4970	74 20 10 117.1
5054-5098	5056-5074	44 20 10 125.8
5122-5208	5140-5168	86 30 15 112.0

WELL 5-3-81-9W5 20 ELONGATIONS 9 BREAKOUTS

3316-3348	3320-3328	32 20 10 61.6
4062-4172	4062-4150	110 90 45 103.1
4208-4218	4208-4216	10 5 159.5
4224-4478	4302-4476	254 100 50 98.0
4490-5126	4856-5098	636 240 120 98.4
5378-5472	5378-5456	94 80 40 97.5
5478-5510	5488-5486	32 10 5 94.1
5530-5600	5530-5578	70 50 25 92.8
5628-5654	5628-5666	26 20 10 93.7

WELL 4-20-81-9W5 28 ELONGATIONS 17 BREAKOUTS

654-1022	786-984	368 200 100 116.0
1084-1116	1094-1112	22 20 10 143.8
1126-1150	1126-1144	24 20 10 15.5
1232-1282	1244-1282	50 40 20 132.0
1310-1628	1350-1602	318 250 125 121.8
1682-1694	1682-1690	12 10 5 116.0
4144-4154	4144-4152	10 5 114.2
4156-4166	4156-4164	10 5 120.7
4188-4234	4190-4208	46 20 10 114.7
4270-4286	4270-4278	26 20 10 103.7
4308-4450	4308-4436	142 130 65 117.5
4466-4506	4546-4584	140 50 25 126.0
4738-4758	4738-4756	20 20 10 60.8
4776-4814	4780-4798	38 20 10 71.8
4878-4882	4882-4980	104 100 50 136.4
4962-5006	4984-5002	24 20 10 146.4
5008-5068	5008-5066	60 60 30 152.7

WELL 14-9-118-8W6 27 ELONGATIONS 4 BREAKOUTS

942-1044	994-1042	102 50 25 147.2
1138-1450	1406-1444	312 40 20 105.4
2260-2286	2260-2278	26 20 10 169.3
2568-2608	2572-2608	40 30 15 153.5



WELL 2-36-118-8W6				10 ELONGATIONS				4 BREAKOUTS				STRATIGRAPHIC INFORMATION
ELONGATED INTERVALS	BREAKOUT INTERVALS	BREAKOUT INTERVALS	BREAKOUT INTERVALS	ELONGATIONS	BREAKOUTS	ELONGATIONS	BREAKOUTS	ELONGATIONS	BREAKOUTS	ELONGATIONS	BREAKOUTS	
4884-4934	50	4884-4922	40	20		142.3						
4964-4978	14	4964-4972	10	5		60.1						
5198-5268	70	5214-5262	50	25		146.4						
5410-5482	72	5422-5466	40	20		166.1						
WELL 9-15-118-8W6												
5164-5286	122	5178-5280e	90	45		128.2						
5356-5386	30	5356-5374	20	10		133.0						
5448-5468	22	5448-5466	20	10		126.2						
WELL A9-5-22-9WS												
5142-5170	28	5148-5166	20	10		76.2						
5206-5222	16	5206-5214	10	5		63.3						
5272-5298	26	5272-5280	10	5		51.2						
WELL 14-21-81-9WS												
5680-5820	140	5680-5782e	80	40		120.9						
WELL 10-9-81-9WS												
5560-5608	48	5560-5598	40	20		123.4						
WELL 3-21-112-6W6												
4384-4414	30	4386-4412	30	15		160.0	----	----	----	SH/LST.		
4650-4664	14	4654-4662	10	5		154.0	----	----	----	DOL/LST		
ELONGATION ENDS*****												
TOTAL DEPTH 5430, KEG RIVER FM.												
WELL 3-1-119-9WE												
924-1800	876	1540-1578	40	20		155.0						
4700-4782	82	4700-4768	70	35		133.1						
5166-5290	124	5186-5214	30	15		54.7						
5722-5752	24	5728-5746	20	10		148.7						
WELL 13-35-112-10WE												
4400-4640	240	4522-4560	40	20		85.0						
4658-4668	10	4658-4666	10	5		144.9						
4844-4884	40	4844-4872	30	15		154.7						
4980-5036	54	5016-5034	20	10		117.7						
5088-5146	58	5088-5136	50	25		164.8						
5190-5300	110	5206-5264	60	30		150.6						
5316-5364	48	5318-5356	40	20		154.4						
5372-5386	14	5372-5380	10	5		158.9						
5524-5542	18	5532-5540	10	5		158.6						
5590-5600	10	5590-5598	10	5		178.5						
5680-5722	42	5680-5718	40	20		162.0						
5750-5774	24	5750-5766	20	10		137.0						
5866-5912	26	5866-5904	20	10		145.2						
WELL 12-33-116-6WE												
790-954	164	902-950	50	25		139.2						
4264-4286	22	4268-4276	10	5		48.4						
4374-4386	14	4374-4386	10	5		121.7						
4500-4520	20	4500-4518	20	10		139.3						
4620-4630	10	4620-4628	10	5		19.5						
4726-4796	68	4750-4788	40	20		56.0						
4856-4868	12	4856-4864	10	5		146.6						
4950-5020	70	4950-4978	30	15		144.3						
WELL 12-33-114-SWE												
4770-4810	140	4777-4905	120	60		119.8						









WELL	5-S-35-11WS	33 ELONGATIONS	12 BREAKOUTS	STRATIGRAPHIC	INFORMATION
ELONGATED	INTERVALS	BREAKOUT	INTERVALS		
14436-14454	18	14441-14449	10		
14462-14474	12	14463-14471	10		
14476-14496	20	14477-14485	10		
14644-14670	26	14659-14666	10		
14702-14784	82	14703-14777e	60		
14850-14884	34	14853-14861	110		
14884-15004	20	14893-15001	10		
15086-15105	22	15089-15097	10		
15940-16056	156	15941-16059	150		
16098-16500	504	16151-16559	440		
16600-16748	148	16603-16711	100		
16802-16868	66	16811-16828	20		
WELL 7-7-86-1W6	30 ELONGATIONS	4 BREAKOUTS			
5578-5600	20	5579-5597	20		
5608-5632	24	5609-5617	10		
5804-5842	38	5805-5833	30		
6306-6624	118	6511-6641	112		
ELONGATION ENDS 6624					
WELL 9-18-83-6W6	12 ELONGATIONS	10 BREAKOUTS			
5128-5138	10	5128-5136	10		
5640-5670	30	5640-5664	10		
5812-5816	354	5812-5816	350		
6178-6200	22	6178-6196	20		
6450-6594	144	6450-6588	140		
6646-6782	136	6646-6774	130		
7252-7288	36	7250-7268	10		
7380-7410	30	7386-7394	10		
7504-7528	24	7504-7512	10		
7566-7570	12	7560-7568	10		
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20		
10932-10952	20	10940-10948	10		
10958-11294	336	10960-11278	320		
11340-11440	100	11346-11424	80		
11456-11550	94	11460-11468	30		
11550-11640	90	11552-11630	80		
11650-11700	50	11658-11696	40		
11700-11856	158	11700-11838	140		
11894-12030	136	11906-12012e	50		
12138-12200	62	12170-12208	40		
12826-16134	308	12850-16118	270		
16244-16290	46	16250-16278	30		
16352-16414	62	16356-16404	50		
16460-16594	134	16462-16580	120		
16634-16652	28	16636-16654	20		
16744-16826	82	16744-16812	70		
16842-16902	60	16846-16894	50		
ELONGATION ENDS 16902					
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20		
10932-10952	20	10940-10948	10		
10958-11294	336	10960-11278	320		
11340-11440	100	11346-11424	80		
11456-11550	94	11460-11468	30		
11550-11640	90	11552-11630	80		
11650-11700	50	11658-11696	40		
11700-11856	158	11700-11838	140		
11894-12030	136	11906-12012e	50		
12138-12200	62	12170-12208	40		
12826-16134	308	12850-16118	270		
16244-16290	46	16250-16278	30		
16352-16414	62	16356-16404	50		
16460-16594	134	16462-16580	120		
16634-16652	28	16636-16654	20		
16744-16826	82	16744-16812	70		
16842-16902	60	16846-16894	50		
ELONGATION ENDS 16902					
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20		
10932-10952	20	10940-10948	10		
10958-11294	336	10960-11278	320		
11340-11440	100	11346-11424	80		
11456-11550	94	11460-11468	30		
11550-11640	90	11552-11630	80		
11650-11700	50	11658-11696	40		
11700-11856	158	11700-11838	140		
11894-12030	136	11906-12012e	50		
12138-12200	62	12170-12208	40		
12826-16134	308	12850-16118	270		
16244-16290	46	16250-16278	30		
16352-16414	62	16356-16404	50		
16460-16594	134	16462-16580	120		
16634-16652	28	16636-16654	20		
16744-16826	82	16744-16812	70		
16842-16902	60	16846-16894	50		
ELONGATION ENDS 16902					
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20		
10932-10952	20	10940-10948	10		
10958-11294	336	10960-11278	320		
11340-11440	100	11346-11424	80		
11456-11550	94	11460-11468	30		
11550-11640	90	11552-11630	80		
11650-11700	50	11658-11696	40		
11700-11856	158	11700-11838	140		
11894-12030	136	11906-12012e	50		
12138-12200	62	12170-12208	40		
12826-16134	308	12850-16118	270		
16244-16290	46	16250-16278	30		
16352-16414	62	16356-16404	50		
16460-16594	134	16462-16580	120		
16634-16652	28	16636-16654	20		
16744-16826	82	16744-16812	70		
16842-16902	60	16846-16894	50		
ELONGATION ENDS 16902					
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20		
10932-10952	20	10940-10948	10		
10958-11294	336	10960-11278	320		
11340-11440	100	11346-11424	80		
11456-11550	94	11460-11468	30		
11550-11640	90	11552-11630	80		
11650-11700	50	11658-11696	40		
11700-11856	158	11700-11838	140		
11894-12030	136	11906-12012e	50		
12138-12200	62	12170-12208	40		
12826-16134	308	12850-16118	270		
16244-16290	46	16250-16278	30		
16352-16414	62	16356-16404	50		
16460-16594	134	16462-16580	120		
16634-16652	28	16636-16654	20		
16744-16826	82	16744-16812	70		
16842-16902	60	16846-16894	50		
ELONGATION ENDS 16902					
WELL 14-21-37-11WS	36 ELONGATIONS	30 BREAKOUTS			
7260-7700	440	7280-7388e	370		
7800-8162	362	7802-8160	360		
8430-8490	60	8432-8490	60		
8564-8622	58	8570-8618	50		
8754-8854	100	8754-8852	100		
8976-8990	14	8976-8984	10		
8990-9038	48	9028-9036	10		
9078-9142	64	9084-9132	50		
9206-9534	328	9316-9530e	160		
9594-10182	588	9622-10180	560		
10194-10240	46	10194-10212	20		
10248-10316	68	10284-10312	30		
10318-10326	508	10330-10768	440		
10838-10894	58	10664-10882	20	</	



WELL 2-29-59-22WS				72 ELONGATIONS				41 BREAKOUTS				STRATIGRAPHIC INFORMATION			
ELONGATED	INTERVALS	BREAKOUT	INTERVALS	ELONGATED	INTERVALS	BREAKOUT	INTERVALS	ELONGATED	INTERVALS	BREAKOUT	INTERVALS	STRATIGRAPHIC	INFORMATION		
7700-7804	104	7702-7796	80	40	108.5	L. CRE.	7752	7752	7752	7752	7752	SS/SH	SS, SH, SILTST.		
7806-7820	14	7810-7818	10	5	86.4	L. CRE.						SS/SH	SS, SH, SILTST.		
7840-8004	164	7862-8000	164	70	164.3	L. CRE.						SILT/SS/SH/COAL	SILT/SS/SH/COAL		
8130-8176	44	8132-8170	40	20	73.0	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8186-8340	154	8190-8328	140	70	134.3	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8366-8496	130	8370-8478	110	55	120.7	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8542-8584	42	8546-8574	30	15	142.7	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8600-8634	34	8600-8618	20	10	5.3	L. CRE.	/8603					SS/SH/COAL/SH	SS/SH/COAL/SH		
8672-8684	10	8672-8660	10	5	125.2	L. CRE.	8603					SS/SH/COAL/SH	SS/SH/COAL/SH		
8714-8730	16	8718-8726	10	5	129.4	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8764-8776	12	8768-8772	10	5	139.3	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8796-8818	22	8798-8804	10	5	148.1	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8826-8838	10	8826-8834	10	5	137.7	L. CRE.						SS/SH/COAL/SH	SS/SH/COAL/SH		
8912-8972	60	8912-8970	60	30	130.0	L. CRE.	/9420					SS/SH/COAL/SH	SS/SH/COAL/SH		
9388-9488	90	9424-9484	40	20	130.0	CRE/MISS	9420					SS/SH/COAL/SH	SS/SH/COAL/SH		
9488-9516	20	9500-9508	10	5	109.0	MISS.	9517					SS/SH/COAL/SH	SS/SH/COAL/SH		
9590-9618	28	9602-9610	10	5	134.8	MISS.	/9635					SS/SH/COAL/SH	SS/SH/COAL/SH		
9622-9638	16	9622-9630	10	5	142.7	MISS.	9635					SS/SH/COAL/SH	SS/SH/COAL/SH		
9686-9708	22	9686-9704	10	5	146.2	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
9758-9806	50	9758-9794	40	20	133.0	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10012-10028	16	10012-10020	10	5	54.2	MISS.	10010					SS/SH/COAL/SH	SS/SH/COAL/SH		
10056-10094	46	10056-10092	40	20	55.0	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10124-10154	30	10128-10146	20	10	93.0	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10162-10178	16	10162-10170	10	5	141.1	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10196-10230	34	10198-10216	20	10	118.8	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10270-10282	12	10272-10280	10	5	107.0	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10340-10392	52	10340-10378	40	20	73.0	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10590-10604	14	10592-10600	10	5	146.8	MISS.						SS/SH/COAL/SH	SS/SH/COAL/SH		
10648-10972	324	10648-10966	320	160	140.1	U. DEVONIAN	10620					SS/SH/COAL/SH	SS/SH/COAL/SH		
10984-11214	230	10986-11204	220	110	139.5	U. DEV.	/11362					SS/SH/COAL/SH	SS/SH/COAL/SH		
11218-11404	166	11228-11378	150	75	136.3	U. DEV.	11362					SS/SH/COAL/SH	SS/SH/COAL/SH		
11444-11482	38	11444-11472	30	15	130.7	U. DEV.	11490					SS/SH/COAL/SH	SS/SH/COAL/SH		
11490-11516	26	11494-11512	20	10	155.5	U. DEV.						SS/SH/COAL/SH	SS/SH/COAL/SH		
11532-11542	10	11532-11540	10	5	124.3	U. DEV.						SS/SH/COAL/SH	SS/SH/COAL/SH		
11552-11572	20	11552-11570	20	10	121.5	U. DEV.						SS/SH/COAL/SH	SS/SH/COAL/SH		
11888-11900	12	11890-11898	10	5	168.4	U. DEV.	11810					SS/SH/COAL/SH	SS/SH/COAL/SH		
12000-12612	12	12000-12608	10	5	156.4	U. DEV.	12379					SS/SH/COAL/SH	SS/SH/COAL/SH		
12656-12746	88	12658-12736	80	40	138.0	U. DEV.	/12677					SS/SH/COAL/SH	SS/SH/COAL/SH		
12794-12826	32	12796-12824	30	15	144.3	U. DEV.	12667					SS/SH/COAL/SH	SS/SH/COAL/SH		
12836-12850	14	12838-12846	10	5	144.5	U. DEV.						SS/SH/COAL/SH	SS/SH/COAL/SH		
12854-12864	10	12854-12862	10	5	132.9	U. DEV.						SS/SH/COAL/SH	SS/SH/COAL/SH		
ELONGATION ENDS 12864, B H L; TOTAL DEPTH 12865, B H L;															
PROD. ZONE LOWER MANNVILLE.															
WELL 10-7-60-14WS				16 ELONGATIONS				11 BREAKOUTS							
7452-7464	12	7454-7462	10	5	115.5										
7558-7600	42	7558-7586	30	15	24.5										
7634-7684	50	7634-7662	30	15	156.0										
7672-7722	50	7686-7714	20	10	123.3										
8034-8054	20	8034-8052	20	10	159.1										
8086-8184	88	8098-8176	80	40	150.5										
8260-8272	12	8260-8268	10	5	165.0										
8284-8374	90	8284-8362	80	40	162.0										
8520-8550	40	8522-8550	30	15	141.3										
8572-8584	12	8574-8582	10	5	150.7										
8650-8660	10	8650-8658	10	5	148.2										



WELL 3-32-34-8WS			15 ELONGATIONS		9 BREAKOUTS		STRATIGRAPHIC INFORMATION
ELONGATED	INTERVALS	BREAKOUT	INTERVALS				
4558-5112	554	4560-4618	58	29	123.0		
4558-5112	0	4640-4858	218	109	155.0		
4558-5112	0	4860-4918	58	29	165.0		
4558-5112	0	4920-5078	158	79	184.0		
4558-5112	0	5080-5112	32	16	129.0		
4558-5112	444	6078-6518	440	220	123.0		
6074-6518	152	6636-6708	72	36	133.0		
6636-6728	0	6720-6750	30	15	135.0		
6636-6728	0	6760-6788	28	14	135.0		
6728-6940	144	6796-6940	144	72	168.0		
6940-7150	100	7100-7148	48	24	115.0		
7150-7400	236	7190-7368	178	89	142.0		
7400-7626	406	7766-7806	40	20	162.0		
7626-7822	780	8078-8120	42	21	142.0		
7822-8722	0	8122-8282	160	80	134.0		
8722-8722	0	8282-8300	18	9	130.0		
8722-8722	0	8310-8716	206	103	127.0		
8716-8902	112	8902-9000	98	49			
8900-9012							
WELL 8-17-37-9WS							
41 ELONGATIONS			39 BREAKOUTS		16 BREAKOUTS		STRATIGRAPHIC INFORMATION
ELONGATED	INTERVALS	BREAKOUT	INTERVALS				
7106-7138	32	7106-7138	32	16	126.0		
7138-7180	26	7156-7176	20	10	126.0		
7180-7316	36	7280-7316	36	18	123.0		
7316-7336	10	7328-7336	10	5	119.0		
7336-7356	10	7346-7352	6	3	125.0		
7356-7396	32	7364-7394	30	15	125.0		
7396-7520	80	7444-7520	76	38	119.0		
7520-7556	40	7530-7556	36	18	118.0		
7556-7590	14	7576-7590	14	7	128.0		
7590-7662	68	7606-7658	56	28	135.0		
7662-7714	20	7698-7714	12	6	120.0		
7714-7774	20	7756-7774	18	4	121.0		
7774-7942	38	7904-7942	38	19	120.0		
7942-8060	114	7946-8060	114	57	117.0		
8060-8164	8	8080-8086	6	3	122.0		
8164-8212	28	8136-8164	28	14	125.0		
8212-8280	26	8186-8212	16	8	127.0		
8280-8370	20	8260-8280	20	10	123.0		
8370-8398	70	8310-8370	60	30	115.0		
8398-8424	14	8380-8398	18	9	120.0		
8424-8474	36	8414-8424	10	5	121.0		
8474-8550	10	8442-8474	32	16	114.0		
8550-8660	24	8540-8550	10	5	130.0		
8660-8776	16	8636-8660	24	12	125.0		
8776-8870	84	8760-8776	20	6	127.0		
8870-8914	30	8866-8866	26	13	130.0		
8914-8966	18	8888-8914	16	8	114.0		
8966-9006	16	8846-8964	18	9	117.0		
9006-9032	6	8950-9006	16	8	116.0		
9032-9212	178	9026-9032	6	3	112.0		
9212-9400	140	9034-9212	178	88	115.0		
9400-9526	24	9250-9398	138	69	126.0		
9526-9622	62	9502-9526	24	12	123.0		
9622-9676	206	9560-9622	62	31	124.0		
9676-10058	132	9676-9878	202	101	117.0		
10058-10194	108	9930-10058	128	64	120.0		
10194-10258	46	10100-10140	40	20	122.0		
10258-10258		10160-10194	34	17	135.0		
		10214-10250	36	18	119.0		



## WELL A2-13-34-11W5 9 ELONGATIONS 7 BREAKOUTS

ELONGATED	INTERVALS	BREAKOUT	INTERVALS	STRATIGRAPHIC	INFORMATION
12940-12958	18	12940-12952	12	MISSISSIPPI. 12771	BELLOY
12964-13010	46	12968-13010	42	MISS.	BELLOY
13000-13240	40	13002-13236	34	MISS.	BELLOY
13560-13582	22	13560-13568	8	13552	PEKISKO
13580-13582	0	13574-13582	8	MISS.	PEKISKO
13700-13740	40	13714-13728	14	MISS.	PEKISKO
13836-13842	6	13836-13840	4	MISS.	PEKISKO
15348-15358	10	15350-15356	6	U. DEVONIAN 15343	DOL/ANHY
			8		DOL/ANHY

## ELONGATION ENDS 15358, CALMAR; TOTAL DEPTH 15497

WELL 7-17-33-7W5	31 ELONGATIONS	17 BREAKOUTS
1586-1716	130	96
1788-1860	72	52
2120-2220	100	24
2120-2220	0	74
2636-2666	30	30
2700-2808	108	106
2814-2850	36	32
2850-2880	30	20
2880-3320	440	28
2880-3320	0	34
2880-3320	0	45
3320-3966	646	60
3320-3966	0	30
3320-3966	0	126
4770-5028	258	126
4770-5028	120	26
5310-5430	678	106
5434-6112	0	44
5434-6112	0	178
5434-6112	0	50
5434-6112	0	21
6150-6388	238	103
6150-6388	0	20
6150-6388	0	142
6150-6388	0	142
6924-7250	326	122
6924-7250	0	133
6924-7250	0	141
6924-7250	0	158
6924-7250	0	147
7310-7510	200	174
7760-7770	30	26
8054-8070	16	8

## BIMODAL WELLS.

WELL 15-9-116-6W6	2 ELONGATIONS	2 BREAKOUTS
4316-4328	12	10
4408-4474	66	30
		5
		15
		86.2
		172.2

WELL 3-1-116-1W6	2 ELONGATIONS	2 BREAKOUTS
4962-4974	12	10
5122-5142	20	10
		5
		146.4
		65.3





TABLE 3.2

Statistics of breakout azimuths for wells listed in Table 3.1  
(All lengths are in feet except where indicated)

WELL	LOGGED	ELONGATIONS	LENGTH	BREAKOUTS	LENGTH	N	MEAN AZIMUTH(DEG)	S.D(DEG)	S.E(DEG)
6-30-46-17W5	3226-12106	80	7148	51	3080	1540	145.9	10.8	0.3
7-9-49-24W5	1418-13938	144	9357	71	2336	1168	156.7	21.3	0.6
12-19-53-8W5	6350-7827	11	420	10	262	131	123.8	2.4	0.2
b-45-A-94-P-14	5650-6089	13	103	6	32	16	60.9	17.1	4.6
6-12-50-11W5 m	461-2874.5m	18	249	12	87	46	126.3	2.9	0.4
14-20-50-12W5	1230-9558	28	831	24	418	209	122.4	4.7	0.3
4-26-51-11W5	8484-9123	2	56	2	52	26	119.8	3.3	0.7
3-32-47-13W5	9566-11494	28	758	23	526	263	131.3	7.6	0.5
6-30-48-12W5	10011-11630	13	316	12	242	121	133.9	4.1	0.4
10-25-45-15W5	10450-11335	13	298	11	204	102	148.3	3.8	0.4
16-16-52-21W5	9000-13452	15	514	13	292	146	124.9	4.6	0.4
10-27-45-16W5	13500-15407	20	2624	14	1342	671	135.3	2.3	0.1
13-9-52-8W5	1526-7420	36	3098	20	1462	730	137.4	7.5	0.3
8-8N-51-9W5 m	2200-2451.5m	11	78	8	30	16	134.7	3.8	1.0
6-20-50-15W5	-----	46	1330	41	976	491	132.5	9.1	0.4
3-36-49-23W5	1507-18002	143	14518	84	3344	1672	151.2	10.6	0.3
5-9-51-9W5	5550-8024	4	2455	2	340	170	147.8	2.1	0.2
10-7-60-14W5	7500-8736	16	450	11	340	170	154.3	8.8	0.7
2-29-59-22W5	7700-12864	72	2717	41	1880	940	135.0	10.4	0.3
14-21-37-11W5	5084-12040e	37	5016	30	3060	1530	151.0	4.7	0.1
7-7-86-1W6	795-7213	30	2453	4	190	95	114.3	3.4	0.4
9-18-83-6W6	5000-7675	12	876	10	710	355	115.4	4.5	0.2
5-5-35-11W5	14205-18000	33	1560	12	940	470	170.7	8.5	0.4
15-29-115-3W6	3500-4644	29	2215	12	690	345	0.6	10.2	0.5
5-28-115-4W6	4368-5205	5	166	3	60	30	153.3	3.6	0.7
12-24-82-6W6	4100-7079	19	1562	10	910	455	105.8	6.4	0.3
11-29-82-5W6	4100-7311	31	1182	19	570	285	104.4	10.7	0.6
3-21-112-6W6	4150-4802	6	146	2	40	20	159.0	0.9	0.2
13-35-112-10W6	4400-6200	22	948	13	360	180	150.8	11.4	0.9
3-1-119-9W6	4650-5774	18	2442	4	160	80	140.4	15.2	1.7
3-11-121-7W6	4800-5624	4	90	2	30	15	160.0	0.0	0.0
12-33-116-6W6	4250-5078	38	2385	8	180	90	140.7	20.8	2.2
14-36-116-6W6	4200-5300	4	146	3	80	40	37.1	13.0	2.1
14-9-118-8W6	648-6091	27	2236	4	140	70	141.3	11.9	1.4
9-15-118-8W6	5042-6142	8	256	3	130	65	128.7	0.8	0.1
2-36-118-8W6	4600-6135	10	628	4	140	70	153.0	9.6	1.2
5-33-109-8W6	692-6024	5	239	3	160	80	103.4	14.1	1.6
7-18RN-109-7W6	4800-5888	12	196	9	140	70	13.0	6.4	0.8
10-17RN-110-9W6	5400-6612	11	1350	5	360	180	47.3	11.5	0.9
2-20-81-9W5	4000-5848	7	620	5	330	165	115.3	0.8	0.1
4-20-81-9W5	622-5855	26	1730	17	990	495	123.1	8.7	0.4
5-3-81-9W5	3200-5871	20	1610	9	620	310	97.8	4.9	0.3
4-15-81-9W5	4900-5927	7	446	3	70	35	116.6	2.5	0.4
A9-5-82-9W5	5100-5590	4	98	3	40	20	67.1	5.0	1.2
3-32-34-8W5	4000-9430	15	3034	8	2020	1010	139.3	7.2	0.2
6-17-37-9W5	7100-10263	41	1866	39	1730	865	121.9	2.9	0.1
A2-13-34-11W5	UNSPECIFIED	9	182	7	128	64	138.3	5.8	0.7
7-17-33-7W5	794-8200	31	3460	17	2204	1102	136.3	13.1	0.4



is given as

$$Y = a_0 + a_1 X \text{ -----3-14}$$

where  $a_0$  and  $a_1$  are obtained from the normal equations

$$\Sigma Y = a_0 . N + a_1 . \Sigma X$$

$$\Sigma XY = a_0 . \Sigma X + a_1 . \Sigma X^2 \text{ -----3-15}$$

which yield

$$a_0 = \frac{(\Sigma Y)(\Sigma X^2) - (\Sigma X)(\Sigma XY)}{N\Sigma X^2 - (\Sigma X)^2}$$

$$a_1 = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{N\Sigma X^2 - (\Sigma X)^2} \text{ -----3-16}$$

$a_1$  is known as the regression coefficient of Y on X. In our case Y represents the azimuth of a breakout and X the depth. To evaluate the significance of a regression coefficient one must consider the scatter or spread of points about the regression line of Y on X. This is given by the statistic *Standard Error Of Estimate of Y on X*. ( $S_{y,x}$ ). Suppose we let  $Y_{est}$  represent the values of Y for given values of X as estimated from (3.14), then

$$S_{y,x} = \sqrt{\{\Sigma(Y - Y_{est})^2 / n\}} \text{ -----3-17}$$

Equation 3-17 can be written as

$$S^2_{y,x} = 1/n . (\Sigma Y^2 - a_0 . \Sigma Y - a_1 . \Sigma XY) \text{ -----3-18}$$

This standard error of estimate has properties analogous to those of the standard deviation in the sense that if lines are constructed parallel to the regression line of Y on X at respective vertical distances  $S_{y,x}$ ,  $2S_{y,x}$ , and  $3S_{y,x}$  from it, it is found (if n is large enough) that there would be



included between these lines about 68% , 95% , and 99.7% of the sample points respectively. Now that a least square regression line has been fitted to the points, we would like to know how well a straight line fits the points or describes the relationship between the variables. In other words we want to determine the goodness of fit of the line fitted to the data. The quantity which does this is the coefficient of correlation ( $r$ ).  $r$  is a dimensionless quantity which varies between  $-1$  and  $+1$ . If  $Y$  tends to increase as  $X$  increases we have a positive or direct correlation. If  $Y$  tends to decrease as  $X$  increases we have negative or inverse correlation. If  $r$  is near zero it means there is almost no linear correlation between the variables.  $r$  can be expressed in several ways but for computational purposes we write

$$r = \frac{N\sum XY - (\sum X)(\sum Y)}{\sqrt{\{N\sum X^2 - (\sum X)^2\}\{N\sum Y^2 - (\sum Y)^2\}}} \quad \text{-----} \quad 3-19$$

It is worth noting that a high correlation coefficient (i.e. near  $1$  or  $-1$ ) does not necessarily indicate a direct dependence of the variables.

### Tests of Hypothesis and Significance

In order for any test of hypothesis or rules of decision to be good they must be designed so as to minimise errors of decision. Obviously this is not simple at all since, for a given sample an attempt to decrease one type of





error is accompanied by an increase in other types of error. However, one type of error may be more serious than another which may call for a compromise in favour of a limitation of the more serious error. The only way to reduce all types of errors is to increase the sample size which may or may not be possible.

### Level of Significance

Rejection of a hypothesis when it should be accepted is defined as a *Type I error*. Acceptance of a hypothesis which should be rejected is defined as a *Type II error*. In testing a given hypothesis, the maximum probability with which we wish to risk a Type I error is called the *Level of Significance* of the test. It represents the probability of our being wrong in rejecting the hypothesis.

The regression equation  $Y = a_0 + a_1.X$  was obtained on the basis of sample data. We are often interested in the corresponding regression equation for the population from which the sample was drawn.

$$Y_{\text{pop}} = A_0 + A_1.X_{\text{pop}}$$

If we wish to test the hypothesis that the regression coefficient of the population  $A_1$  is zero at some chosen significance, we use the fact that the statistic  $t$  has Student's distribution, with  $(n-2)$  degrees of freedom, where

$$t = \frac{(a_1 - A_1).S_x}{\sqrt{S_{y,x}^2 / (n-2)}} = \frac{(a_1 - A_1).S_x}{\sqrt{\{(n-2) / (1-r^2)\} S_{y,x}^2}} \quad 3-20$$

Here  $S_{y,x}$  is the standard error of estimate of  $Y$  on  $X$ ,  $S_x$  is the standard deviation of  $X$ , and  $r$  is the correlation coefficient. This fact can be used to find the confidence



intervals for population regression coefficients from sample values.

In applying Equations 3-14 through and including Equation 3-20 to our data we can then compare our  $t$  values to the Student's  $t$ -distribution table (given in many statistical text books), with the correct number of degrees of freedom, and find at what level of significance the hypothesis is to be accepted or rejected.

Table 3.3 presents regression results for 21 wells in which breakouts were measured over a depth range greater than 2000 feet. Column 3 shows values for the regression coefficient  $a$ , for the sample and column 4 gives the standard error of estimate of the azimuths on depths. Column 6 shows the various  $t$ -distribution values and column 7 shows the degrees of freedom which can be used to find the level of significance at which the test can be accepted by reading the appropriate  $t$ -value and degree of freedom from tables in text books. In column 5 we have the values of the correlation coefficient.

### Results

It is clear that the regression coefficient  $a$ , is in all wells small enough to have arisen with more than 5 percent probability, in a population having  $a=0$ . Of the 21 wells listed in Table 3.3, 10 have  $a > 0$  and 11 have  $a < 0$ . We cannot reject the hypothesis that the azimuths of the breakouts are unrelated to depth and the variation observed with depth is only by chance



TABLE 3.3

Regression of breakout azimuths on depths showing the regression coefficients  $a_1$  (deg/ft), the correlation coefficient  $r$ , the standard error of estimate  $S_{y,x}$ (deg), the  $t$ -distribution, and the degrees of freedom  $df$ . (Only wells with the entire breakout intervals extending through 2000 feet depth intervals are listed here).

Well	$a_0$ (deg)	$a_1$ (d/ft)	$S_{y,x}$ (deg)	$r$	$t$	$df$
6-30-46-17W5	155.57	-0.00232	28.3	-0.183	-0.059	619
7-9-49-24W5	116.95	0.00112	45.7	0.087	0.038	1121
14-20-50-12W5	129.78	-0.00080	10.8	-0.052	-0.011	199
10-27-45-16W5	175.12	-0.00290	4.1	-0.460	-0.085	680
13-9-52-8W5	143.37	-0.00242	19.3	-0.203	-0.050	405
6-20-50-15W5	281.75	-0.01416	18.1	-0.475	-0.345	459
3-36-49-23W5	143.06	0.00042	28.7	0.062	0.018	1757
5-3-81-9W5	95.82	0.00046	10.5	0.024	0.008	308
4-20-81-9W5	122.32	0.00055	18.9	0.050	0.012	493
3-1-119-9W6	172.69	-0.01128	30.8	-0.489	-0.114	78
12-33-116-6W6	144.05	-0.00677	41.2	-0.269	-0.066	88
11-29-82-5W6	35.66	0.01150	20.2	0.356	0.200	263
15-29-115-3W6	153.81	0.00856	20.3	0.304	0.166	343
5-5-35-11W5	-289.62	0.02845	19.1	0.716	0.881	468
9-18-83-6W6	164.52	-0.00786	10.6	-0.306	-0.154	348
14-21-37-11W5	151.93	-0.00011	8.1	-0.037	-0.005	1838
2-29-59-22W5	113.99	0.00178	22.9	0.115	0.055	938
3-32-34-8W5	167.45	-0.00442	11.9	-0.446	-0.158	1022
6-17-37-9W5	130.71	-0.00105	5.5	-0.176	-0.031	863
A2-13-34-11W5	65.50	0.00553	14.5	0.194	0.044	62
7-17-33-7W5	72.14	0.01084	31.9	0.533	0.420	1073





#### IV. BREAKOUTS AND STRESS ORIENTATIONS IN THE WESTERN CANADIAN SEDIMENTARY BASIN

##### Relation of Breakouts to Location and Lithology

In Chapter III the statistical distribution of breakout azimuths has been set out. In most holes, azimuths are closely grouped together, as shown by the mean orientations of the various breakout zones within each hole (Table 3.1), and the standard deviations listed in Table 3.2. In the holes logged through a sufficient range of depth to allow investigation of regression of azimuth on depth, such regression is not significantly different from zero (see Table 3.3.). Figure 18 is a map showing how the azimuths change with location. The azimuth referred to is the representative azimuth of breakout for the well as a whole obtained by combining the various breakout azimuths in the well given in Table 3.2. The central dot shows the position of the hole and the mean orientation of the breakout is shown by the line through it. The number attached to the line is the mean orientation of the various sections in the well.

We now consider the relation of the breakout azimuth to the geological age (i.e. period), formation and lithology. These data are shown in Table 3.1 under the columns Formation (Fm) , Lithology (Lith), and Period (Age), and have been obtained from the sample descriptions for the wells. Sections under these columns which are blank indicate that the information was not available for those wells. It









is evident, however, from these columns that breakouts have occurred throughout the sedimentary basins in rocks ranging from Devonian to Cretaceous in age. In some cases breakouts have been observed at different depth intervals running through the entire depth of the hole.

A look at the columns headed formation and lithology reveals many interesting points. Breakouts have occurred in formations like Elk Point, Wabamun etc. in the Devonian right to Belly River and Bearpaw formations in the Upper Cretaceous period. Within a given formation some sections show breakouts but other sections do not, for reasons which are obscure. Breakouts may occur in any position such as the top, middle or bottom sections of the formation. The pattern does not appear to be regulated by the type of the lithology. A word of caution must be sounded here. The sample descriptions from which our stratigraphic information was obtained merely stated the predominant lithologies found and only stated the traces present but not the percentages of these traces. A knowledge of the percentages of such traces would be useful in view of the fact that there are several diagenetic processes involved in the transformation of unconsolidated sediments to consolidated sedimentary rocks. However, there are formations in which only one kind of lithology, without any traces, runs through the entire depth range in which breakouts occur in certain depth intervals separated by uncaved intervals.



Although the elongated sections which did not satisfy our three criteria for selecting a breakout are not included in Table 3.1, a careful study of the ages, formations, and lithologies of these zones was carried out. The entire study shows that breakouts are not lithologically controlled. They have occurred through most parts of the Western Canadian sedimentary basin cutting through limestone, dolomite, shale, sandstone, anhydrites, cherty shales, siltstone, and varying combinations of these sedimentary rocks. At the same time, the intervening zones without breakouts have exactly the same types of lithology as already listed. It can therefore be safely concluded that the breakouts and/or elongated zones are not significantly related to the lithology and age of formation. There is no evidence of any significant relation between breakout azimuths and the properties of the rock.

This agrees with the observation of Babcock (1978) on a smaller sample of breakout data. The logs used in this study give no indication of the dip angles of the various strata. This study therefore cannot add to the evidence given by Cox (1970) and Babcock (1978) that the breakouts and their azimuths are unrelated to the dip.

There are two wells included in this study in which the two breakout zones selected using the three criteria have azimuths approximately orthogonal to each other. These wells are included in Table 3.1. Some other wells have one or two intervals with breakout azimuths differing by as much as  $90^{\circ}$





from the dominant azimuths. Such wells have not been referred to as bimodal wells. This study has shown that whatever their cause, a predominant direction of breakout is almost always present. Variations in this direction with position may represent local variations in the orientation of the horizontal stresses.

Pore water pressure  $P_w$  and the pressure  $P_m$  in the drilling mud which fills the hole were also disregarded in the discussion in Chapter II. These have no effect on the orientations of the fractures (Gough and Bell, 1982) nevertheless, they modify the magnitude of the stress differences. At points P and Q (p, q) of figures 12 and 13

$$S_r = P_m - P_w \text{ and the tangential stress is}$$

$$S_\phi = 3S - s - P_w \text{ and } T_{r\phi} = 0.$$

Thus the stress difference becomes

$$S_\phi - S_r = 3S - s - P_m.$$

The Mohr circle Fig. 10 contracts in diameter by  $P_m$  and moves to the left by  $P_w$ .  $P_w$  is often lithostatic and cannot be less than hydrostatic, so that  $P_m$  may often be comparable to  $P_w$ . The shear strength  $T_0$  required to resist failure will be reduced somewhat as a result of the shift of the Mohr's circle to the left and the nature of the envelope. The shift of the circle to the left implies a reduction of the stresses, in a biaxial stress field, needed to cause failure.



## Hydraulic Fracturing

In-situ measurement of stress by hydraulic fracturing can both provide the orientation and magnitude of the principal components of stress, if  $S_3$  is not vertical (Kehle, 1964 ; Fairhurst, 1964), provided the fracture pressure, the instantaneous shut-in pressure (ISIP), the pore pressure and the tensile strength of the rock are measured. Hydraulic fracturing is a petroleum engineering technique carried out at depths of several thousand feet to stimulate production of oil bearing horizons. An understanding of the basic principles which govern the development of the fractures is a prerequisite to its successful application. It is assumed that the hole is drilled parallel to one regional principal stress usually the vertical stress  $S_v$ , which is assumed to be equal to the lithostatic load  $\rho \cdot g \cdot h$ . The stresses  $S$  and  $s$  normal to the hole axis are then the other two principal stresses and are horizontal. The pre-existing regional stresses produce stress concentration close to the borehole with the maximum value of  $(3S-s)$  and a minimum of  $(3s-S)$  in the tangential direction. The smallest tangential stress of  $(3s-S)$  at the ends of the diameter parallel to  $S$  may be greater than, or equal to, or less than zero depending on the  $S/s$ -ratio. If a section of the hole is sealed and pressurized to a pressure  $p$ , a second stress system will be superimposed on that just described. In the region close to the packers, Kehle (1964) assumed that the packers are held in place by a band of



uniform shear stresses and made an analysis to determine these stresses. His results indicate that high tensile vertical stress of the order of the applied pressure ( $-p$ ) is induced in the immediate vicinity of the packers, and in about 80% of the central region of the interval a tangential tensile stress is generated. If however inflatable packers are used to seal the hole a normal pressure will be exerted resulting in a considerable reduction of the axial tension and will require a much higher pressure for the initiation of horizontal fractures. Before such high pressures are reached a fracture will be initiated vertically (Zoback et al, 1977). As the pressure in the hole is increased a fracture will be initiated at the wall of the borehole when the tension generated by the fluid is sufficient to overcome the combined effect of the regional compression ( $3s-S$ ) and the rock strength. The plane along which a fracture will commence will be that across which the resultant of the effective compressive and tensile strengths are first reduced to zero. In the case of a smooth cylindrical wellbore this plane must be vertical and perpendicular to the least principal regional stress. If the vertical stress is the minimum principal stress a horizontal tensile fracture might be expected to form (Kehle, 1964; Hubbert and Willis, 1957) However, Zoback et al (1977) have argued against this. This contention of Zoback et al is supported by laboratory experiments reported by Haimson and Fairhurst (1970). Haimson (1976b) suggested that if  $S_v = S_3$  then





induced fractures initiate in a vertical plane and then become horizontal as they propagate a few hole radii from the wall into the undisturbed regional stress field.

The breakdown pressure  $P_b$  is recorded. If the pump is shut off immediately and the hydraulic circuit is kept closed an instantaneous shut-in pressure, a pressure that is just sufficient to hold the fracture open, is recorded. The theory derived by Hubbert and Willis (1957) and Kehle (1964) for fracture around a pressurized borehole is used to relate  $P_b$ , the breakdown pressure, the instantaneous shut-in pressure ISIP, to the in-situ principal stresses and the tensile strength,  $T$ , of the rock. For a vertical borehole, the tensile fracture should be oriented in a direction perpendicular to  $s$ , the least horizontal principal stress, at least within the immediate vicinity of the borehole, and the magnitude of  $s$  is equal to ISIP.  $S$ , the maximum horizontal principal stress, is determined from the equation

$$P_b = 3s - S - p + T \text{ -----4-1}$$

where  $p$  is the static pore pressure in the rock surrounding the borehole.  $T$  can be measured in the laboratory and  $p$  in the field. From the discussion so far it is clear that the hydraulic fracture will generally take place parallel to the maximum horizontal principal stress. Breakouts on the other hand occur parallel to the lesser horizontal principal stress or perpendicular to the larger horizontal principal stress. It is therefore, not surprising that fractures detected by the 4-arm dipmeter tool may occasionally show





two trends one being a major one, if mud pressure has occasionally produced inadvertent hydraulic fracturing.

The question to be asked will therefore be, how the hydraulic fracturing comes into play? Prior to the deliberate application of hydraulic fracturing by man, its occurrence has been detected during water flooding and squeeze cementation processes. Rocks have been accidentally fractured during grouting treatments and when high specific gravity muds have been used in drilling wells. Furthermore, it has been observed (Von Schonfeldt and Fairhurst, 1969) that formations without any measurable permeability under natural conditions have a noticeable effect of rate of loading. At a low rate of application of pressure, the formation breakdown is lower than at a higher loading rate. Such a loading rate may be likened to the varying rate at which drilling is done depending on the commercial importance of the zone being drilled. Accidental hydraulic fracturing during the drilling stage with high specific gravity mud can therefore not be ruled out. It is worth mentioning also that although the vertical stress approximates the overburden, thus implying a reduced effect of accidental hydraulic fracturing, we should not lose sight of the fact that the oil wells are restricted to sedimentary layers. Sediments are sheet-like deposits of various competency stacked one upon another. Layered sequences of unlike materials transfer stress much differently than homogeneous and isotropic materials, a phenomenon known as



*arching*. Arching may also involve the reaction with time between beds of different competency and hence disrupts the uniform stress distribution with depth which is assumed under homogeneous and isotropic conditions. Such an effect creates regions of both high and low stress concentrations. As a result of the alteration of the stress distribution by arching we can expect that in the same oil field, for a given horizon breakdown pressure can vary significantly as evidenced by the results obtained in the Gulf Coast area where vertical fractures have occurred at injection pressures far less than the total overburden pressure ( Scotts, Bearden, and Howard, 1953).

Another point which may need examination is the assumption that one principal stress is vertical and that the borehole is drilled parallel to this regional principal stress. The assumption that the hydraulic fracture propagates perpendicular to the least principal stress ( Hubbert and Willis, 1957) has been supported by agreement between hydraulic fracturing , geological and seismologically determined stress field indicators (Zoback and Zoback, 1980 ) However, McGarr and Gay (1978) found that in South Africa the directions of the most nearly vertical principal stress fall within a circle of  $30^{\circ}$  about the vertical. They also mentioned that stress measurements in deep mines in Canada, the United States and in Australia support significant departures from the assumption that one of the principal stress directions is vertical. This implies



that the two initially assumed horizontal stresses are not truly horizontal. Recognition of such a fact will also help in accounting for the irregular nature of fractures.

#### **Breakout and Stress Orientations in Western Canada**

Figure 19 shows the directions of the long axes of breakouts (or the direction of the least principal horizontal stress) and figure 20 indicates the inferred orientations of the maximum horizontal principal stresses for the results presented by Babcock (1978), by Gough and Bell (1981) and in this study, for the West Canadian Sedimentary Basin. It can be seen from this map that breakouts can serve as a useful method for mapping the various tectonic stress zones based on the stress trajectories in any given locality where directional spalling of the walls of a well are observed. The absence of significant variation of azimuths with depth, wherever these phenomena are observed, may imply that such large horizontal principal stresses are not necessarily limited to the top crustal sedimentary rocks but may be continued down into the deep and older Precambrian igneous rocks of the earth's crust





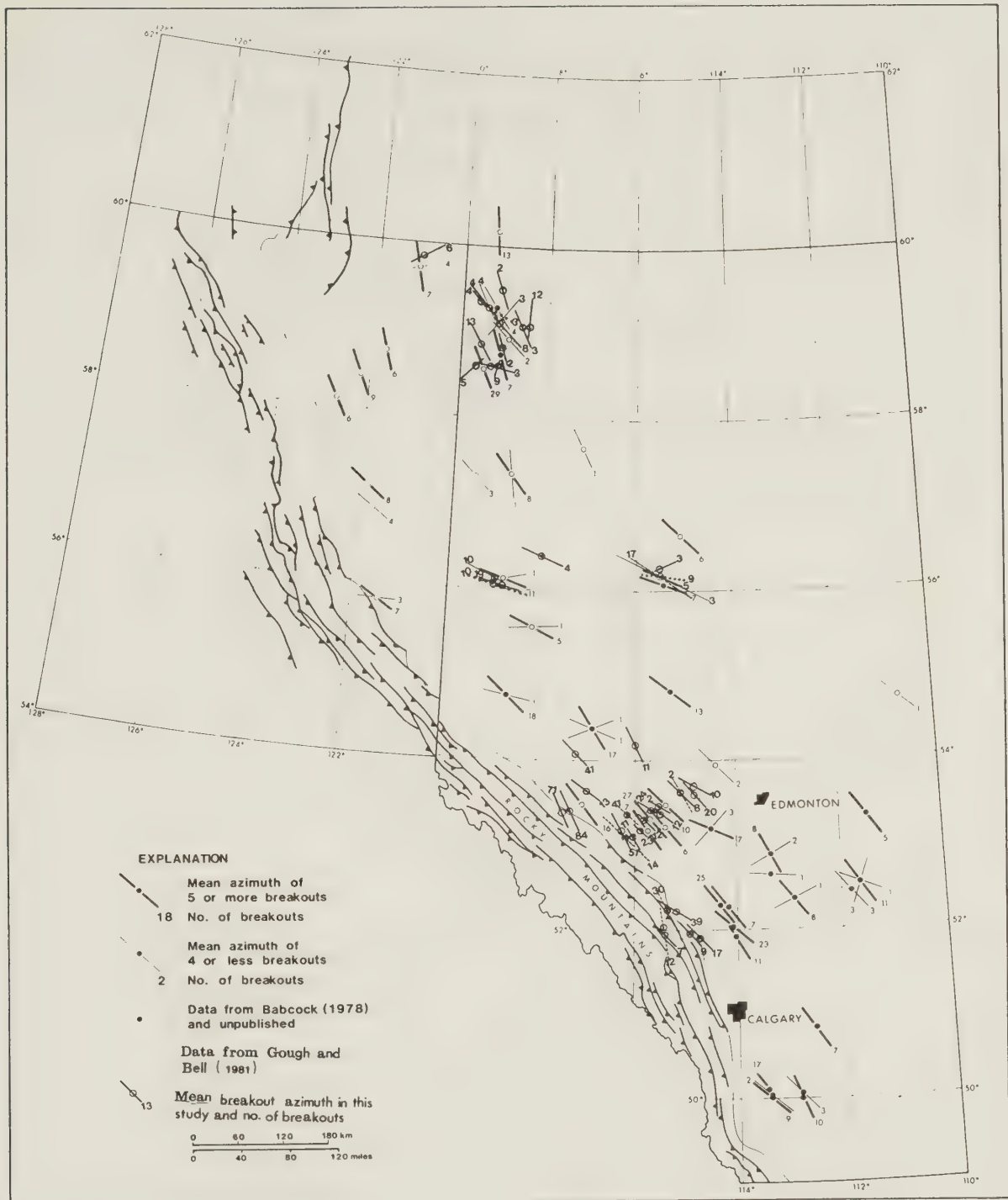


Figure 19.... Map of Alberta showing the orientations of the smaller principal horizontal stresses as inferred from the breakouts



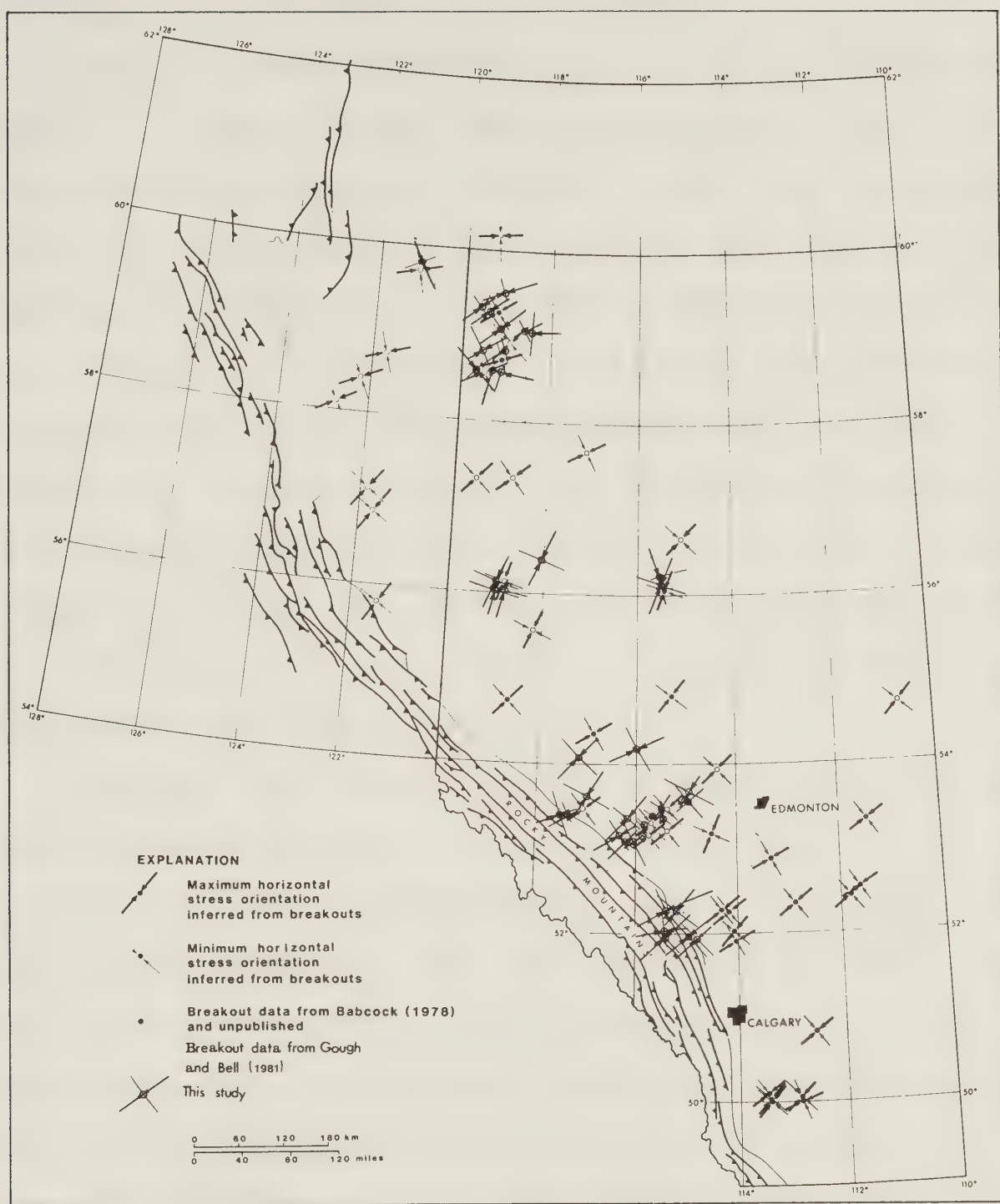


Figure 20.... Inferred directions of the larger horizontal principal stresses for the results presented by Babcock (1978), Gough and Bell (1981) and in this study



## V. DISCUSSION

### The Physical Significance of the Breakouts

Babcock (1978) concluded on the basis of his joint-sets study in Alberta (1973) that breakouts are a result of the drill encountering steeply dipping zones of pre-existing faults or joint-sets, which may or may not be open. Preferential breakage, in his view, is then controlled by a near-vertical joint which intersects the well and which in carbonate rocks may be solution-widened. In his view, the parallelism between azimuths of elongation and regional joint sets in Alberta and the consistency of the azimuths of elongation within, and between wells in varying lithologies all indicate an origin caused by the drill encountering a fracture or zone of fracture.

However, his study of joint-sets revealed four concentrations of joint directions on the surface, (NW, NE, N, and E). On a non-polar or axial data basis these four joint-sets directions may be represented as NW-SE, NE-SW, N-S, and E-W. Babcock's study of breakouts showed only one significant concentration of subsurface breakout direction that is NW or NW-SE. Bell and Gough (1979) argued that on Babcock's hypothesis the other joint directions would be expected to be represented in the breakout azimuths, but they are not. In addition Bell and Gough doubted that the joint directions at the present surface would necessarily be parallel to those at depths in excess of 2 km at which breakouts occurred, in view of the fact that sediments





involved in his study might have been laid down in varying tectonic settings between Devonian and Cretaceous times and that joints in general arise in several ways; such as soon after deposition of a sedimentary rock, or during later deformation, or as it undergoes erosion. These facts led Bell and Gough to put forward an alternative hypothesis to better explain the cause of breakouts and, perhaps, also the cause of surface joints of Babcock's System 1 (NE and NW strikes). They therefore proposed that in a general stress field, with large and unequal horizontal stresses, the holes themselves concentrate the stresses so as to produce subsurface breakouts with the breakout azimuth parallel to the smaller horizontal stress and normal to the larger horizontal stress. The analysis of such stress concentration by a circular hole was first found by Kirsch and has been discussed in chapter II.

From the present study, it is observed that the significant breakout azimuth or the preferred breakout orientation is SE, that is NW-SE when expressed axially. Thus only one of the four surface joint directions is significant, although a few breakout zones tend to show the NE or NE-SW strike. In short only the NW-SE set of Babcock's System 1 of surface joints is observed as breakouts. On Bell and Gough's hypothesis that breakouts are stress controlled, the observed irregularities have been fully accounted for. Babcock's hypothesis fails to explain why breakout azimuths do not exhibit the other surface joint directions. Babcock's





study also revealed that breakouts are unrelated to depth, lithology and the age of the formation. It is further observed by Babcock and in this study that breakouts are discrete, beginning and ending sharply, independently of depth, lithology and age of formation. It is very difficult to understand how breakouts can begin and end sharply, within and between formations, separated by similar lithologies with no regard to depth. In view of the various diagenetic processes involved in the transformation of the deposited sediments to consolidated sedimentary rocks from the Devonian to the Cretaceous age, which is more than  $10^8$  years, one wonders how surface joints can be a reflection of subsurface joints. In fact factors like cementation, compaction, recrystallisation, and many other processes involved in diagenesis coupled with the high temperatures and duration of the tectonic stresses will modify the response of the rock to stress, while any failure remains brittle. Such variables may lead to failure in some intervals in the sedimentary column, whereas the same material in adjoining area may not fail under the existing stress field. Consequently Bell and Gough's hypothesis can account for this discrete nature of breakout occurrence in wells. The presence of pre-existing fractures would also result in enormous circulation loss, which is however very small in this region. It is known that extensive circulation loss often leads to formation damage. Such damage often requires that hydraulic fracturing be carried out, this time



not just as a means of providing an increased permeability path into the otherwise tight formation, but as a means of creating a conducting channel through the damaged zones into the virgin formation. The absence of such effects as high circulation loss, damaged formation resulting from the drilling fluid escaping into the formation, and the complete absence of massive hydraulic fracturing processes in Alberta only go to support the point that pre-existing fractures of size as implied by Babcock are unlikely in the subsurface of Alberta. The only hydraulic fracturing process reported is the one at West Pembina (MacLeod, 1977). The fracturing direction is in agreement with the known theory on hydraulic fracturing and thus lends support to Bell and Gough's hypothesis (Gough & Bell, 1981). It is therefore, the present author's opinion that in the absence of positive extrapolation of surface joints to subsurface ones, Babcock's hypothesis does not account for the azimuthal alignment of breakouts. It is therefore concluded, subject to comparison with other stress measurement techniques in future, that breakouts are a result of the stress concentration by the borehole, in a general stress field having large and unequal horizontal stresses, at the walls of the hole resulting in the failure of some of the rocks. This is the hypothesis postulated by Bell and Gough.

Based on the Bell-Gough hypothesis, we find that the greatest horizontal principal stress trends N 25° E to N 60° E, with the majority occurring around N 42° E, and that the



least principal horizontal stress is directed N 30° W to N 65° W, with the majority around N 48° W throughout most parts of Alberta. Sbar and Sykes (1973) have reported the greatest principal horizontal compressive stress to be trending East to Northeast in Eastern North America and in the intermountain seismic belt in Montana, Idaho and Utah the minimum compressive stress trends East to Southeast. Our result thus fits closely to the general pattern of principal stress orientations reported in the older craton of the North American continent.

### **Implications of the Study**

Measurement of stress by in-situ techniques can provide both the orientations and magnitudes of the principal components of stress. Such measurement of the stress field within the crust can provide, perhaps, the most useful information concerning the forces responsible for various tectonic processes, such as earthquakes. For example Sykes et al (1972) reported the occurrence of a series of small earthquakes generated in 1971 by the high pressure injection of fluids in a salt recovery well near Dale, about 10km east of Attica in Western New York State. When the high pressure injection ceased seismic activity dropped in three days from a rate of about 100 events per day to a rate of a few events per month. The triggering of the earthquakes was attributed to the increase of fluid pressure in the rocks under high tectonic stress. Healy et al (1968) also reported the Denver earthquakes. The Rocky Mountain Arsenal disposal well, which







was drilled through 3761 metres of sedimentary rocks of the Denver Basin near the city of Denver, Colorado, was intended for the disposal of waste fluids. The injection of fluid which was started from March 1962 to September 1963 was at a rate of about  $2.1 \times 10^7$  litres per month. Fluid injection ceased from October 1963 to August 1964, but was resumed under gravity from September 1964 until March 1965 at a rate of about  $7.5 \times 10^6$  litres per month. The rate was then increased to about  $1.7 \times 10^7$  litres per month until February 1966 when the exercise was terminated because of growing evidence of its connection with local earthquakes. Evans (1966) used the epicentral locations by Wang (1965) of earthquakes near the well and showed a correlation between rate of fluid injection and the earthquake frequency, with a lag of the earthquakes in the order of a few weeks behind the injection. The events which initially had epicentres less than 6 km from the well, were observed to continue with migration of the foci to the northwest. Healy et al examined the probability of a chance association, in both time and place, between the fluid injection and the earthquake swarm on the basis of the seismicity of the region and estimated this probability to be about 1 in 2.5 million. Healy et al (1968) therefore explained both the occurrence of the northward migration and the occurrence of the three large earthquakes, among a series of earthquakes, of magnitude  $\geq 5.0$  to the outward diffusion from the well of a pressure front in the pore water. This diffusion would imply that the



volume of rock affected by the increased water pressure likewise grows and this increased volume is thus associated with the the large earthquakes in a large volume of major strain. But it is well known that the increase of the pressure of the water in pores and cracks produces no shear stress and therefore cannot cause an earthquake. The induction process of the observed earthquakes was attributed to the presence of a nearly critical initial stress which could eventually cause failure of rock by triggering through by increase of pore water pressure as the Mohr circle is shifted to the left. This explanation is based on the Mohr-Coulomb failure theory, and the Hubbert and Rubey hypothesis of effective stress (1959) ( Healy et al, 1968; Gough and Gough, 1976; Gough, 1978).

The Rangely Experiment carried out by the United States Geological Survey at the Rangely Oilfield in Northwestern Colorado (Raleigh et al, 1972, 1976) was designed to verify quantitatively the explanation of the induced seismicity at Denver. There was an excellent agreement with the hypothesis. Raleigh et al (1976) also pointed out that this verification might have implications for the control of natural earthquakes at least in cases where it might be possible to pump water in and out of an active fault. This could possibly relieve the stress by first reducing the water pressure in wells at two points to strengthen the fault at those points, and then finally triggering an earthquake between the two points by injecting water there.



Knowledge of the state of stress is also critical to the design of underground excavations for mining and for nuclear waste disposal ( Jaeger and Cook, 1969; Gough, 1978). The massive hydraulic fracturing of formations in oil and gas fields to stimulate production is another field for which knowledge of the stress field at depth is very important. Application of stress measurements to the solution of problems in tectonics is not as straightforward as in engineering problems. The engineer is concerned with the stress field affecting the rock, whereas the geophysicist or geologist attempts to deduce the processes that might have caused the stresses. Thus the importance of the knowledge of principal stress orientations cannot be overemphasized. The use of breakouts to add principal stress orientations to the existing information on seismicity and tectonics will significantly increase the data base relevant to intra-plate earthquakes. Maps such as that shown in Figure 20 of the orientation of the horizontal principal stresses, will be of great value to engineering purposes and in addition will have important implications for the geophysics and structural geology of the area.

It must be emphasized that any attempt to relate the stress field to tectonic processes, must be based on a correct interpretation of the stress field. A variety of postglacial geologic features such as folds, faults, pop-ups, and other rock squeeze indicators may show high compressive stress in a region. Residual stresses may exist





in a rock according to its history of processes such as burial, lithification, denudation, heating, cooling, and past tectonic events and may persist to some extent after the rock is freed of boundary loads (Friedman, 1972). Such residual stresses may complicate the interpretation of stress observations. However, it would be difficult to attribute a consistently oriented stress field over a large area, as in the entire Western Canadian sedimentary basin, to residual stresses. The observed stress field almost certainly reflects tractions now acting on this part of the North American plate. The recent review by Zoback and Zoback (1980) shows that northeast-southwest horizontal larger compressions dominate the older craton of North America, as in the Northeastern United States (Sbar and Sykes, 1973) and the Western Canadian data fit well in this overall picture.

A knowledge of the orientation of the horizontal stresses is of great importance in the planning of a hydraulic fracture program. Near-surface ore bodies can be economically mined by open pit methods. Below the practical depth of open pit mining, new mining methods must be sought and developed to make deep ore bodies available to economic extraction. Solution mining has become a major method of extracting subsurface evaporite minerals because it is both economical and efficient. Hydraulic fracturing, apart from the fact that it is the only method currently in use that enables measurement of stress to be made at large distances





from a free surface, is used to develop permeable surface areas exposed to solvent for solution mining. Thus cross-connection of salt wells (and perhaps oil wells in future) by means of extending a hydrofracture from one well to the other has become a boon to the salt industry (Pullen Jr., 1958). The two wells so connected may be referred to as source and target wells, with the pressurized well from which the hydrofracture originates as a source. The target well acts in one of two ways. Ideally the target well should act like a sink so that the propagating fracture will want to intersect it. In some cases however, with the high stress concentrations around it, it may act as a barrier to the approaching fracture and most target wells act like minor sources, and thereby deflect the propagating fracture around themselves. To overcome this secondary role of the target well several techniques proposed have had promising effect. It will always be good practice to produce hydraulic fractures from the target well either before or during the major fracture attempt from the source, (Bays et al, 1960). Back fracturing has proved to be successful in several cases where a fracture connection was not successful in the original direction. It is also thought that much of the stress concentration around the target well might be reduced, if the target well were pressured up and held during the fracture operation of the source well, at a pressure less than the breakdown pressure. Crawford and Collins (1954) point out the adverse effects on the



efficiency of a fluid injection project when injection and production wells are not oriented with the natural and induced fractures. If wells are properly oriented and fractures of adequate length are induced, radial flow is reduced and areal-sweep problems are minimised.

We now consider the attempt to look critically at the lithologies in this study. The phenomenon of stress redistribution at depth in layered sequences of unlike rocks, known as arching, has been mentioned earlier. Arching may also be visualised in terms of massive competent beds acting as beams to carry the load imposed by crustal stress field and thereby reduce the stress on the less competent beds. Such an effect is important to the design of any subsurface opening. Perhaps the greatest importance of arching is its influence through the stress distribution on the breakdown pressure. The object of a multiple fracturing process (multifrac) is to obtain greater fracture area through closer fracture spacing, than could be obtained from the same number of wells with single vertical fractures. This method requires substantial deviation of the wells from the vertical and, therefore both the measured well length and drilling cost per unit length are increased when compared with a vertical well. Theory predicts a vertical hydraulically induced fracture in a strike-slip stress field, and such observations as are available confirm that in the Alberta sedimentary basin hydraulic fractures are generally vertical (Macleod, 1977 ). Thus it should be



possible to generate several fractures from a suitably deviated hole. Hydraulic fracturing theory predicts that fractures will propagate in a direction normal to the least principal in-situ stress or parallel to the maximum principal stress. Some experiments with non penetrating fluid indicate fractures may at least initiate parallel to the axis of the wellbore. A fracture initiated from an inclined open-hole wellbore might then result in a configuration unsuitable for the development of multiple vertical fractures. However, by casing the wellbore and perforating the casing over a small depth range, fracture initiation can be made to occur from a point or a small spherical cavity (Strubhar et al, 1975). The orientation should then be governed by the in-situ stress condition. Strubhar et al have shown experimentally that if the fractures are indeed oriented as dictated by in-situ stresses, the fractures will be sufficiently parallel and therefore be effective. The productivity index they found was two and three times that of a conventionally fractured well in the field.

## Conclusions

It has been argued earlier in this chapter and elsewhere that the consistency of the breakout azimuths irrespective of depth, lithology and age of the formation in almost all the wells, and the fact that the breakout azimuths reflect only one out of the two joint systems of Babcock, lead to a safe conclusion that the breakouts are







stress controlled or possibly both.

It is probable that some of the deviations from the generally northwest-southeast orientation of the breakouts can be attributed to local stress anomalies and some also to accidental hydraulic tensile fracturing during drilling with high-density mud. Zoback and Pollard (1978) noted that although high viscosity fluid reduces fluid penetration into pre-existing fractures or into permeable rocks, it has the disadvantage that such fluid may lead to hydraulic fractures significantly in advance of the observed breakdown in the pressure-time history.

On the hypothesis that breakouts are stress controlled, the following conclusions can be made.

1. The state of stress in the earth's crust is not in general lithostatic. There are high horizontal stresses relative to the vertical stress irrespective of depth. Tectonic conditions and the various sedimentary layers may, however, influence this state of stress.
2. Perhaps, some horizontal fractures were wrongly taken for vertical ones. Such horizontal fractures are believed to result from local inhomogeneities like bedding planes, joint-sets and the phenomenon of arching.
3. Although breakouts tend to occur in preferential directions dictated by the orientation of the regional stresses, induced vertical fractures may also be expected to have a preferential direction. In most cases



this direction is not identical to that of breakouts, but at right angles to them.

### Concluding remarks

The consistency of the data in this region of North America and elsewhere on the continent strongly suggests that this measurement does not merely reflect local stresses but does, in fact, reflect a regional pattern. The existence of high stresses is not confined to any member of the rock formations, nor to any particular age of the formations. The phenomena of high stresses exist elsewhere in the North American continent and the world (Sbar and Sykes, 1973; Hast, 1973,1974; Lo, 1978; McGarr and Gay,1978; Gough, 1978; Zoback and Zoback, 1980) as evidenced by features like breakouts, folds, postglacial faults,and pop-ups. These stresses are probably related to the current tractions on the North American plate in the overall context of plate tectonics.



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## APPENDIX A

### A. Properties Of $\bar{\varnothing}$

Let  $\varnothing_1', \dots, \varnothing_n'$  be the angles obtained from  $\varnothing_1, \dots, \varnothing_n$  when the new zero direction is  $\alpha$ . Let

$$\bar{C}' = 1/n \cdot \sum \cos \varnothing_i' ; \bar{S}' = 1/n \cdot \sum \sin \varnothing_i' \text{-----A-1}$$

We have  $\varnothing_i' = (\varnothing_i - \alpha) \bmod 2\pi$ . So that

$$\bar{C}' = \bar{R} \cdot \cos(\bar{\varnothing} - \alpha) ; \bar{S}' = \bar{R} \cdot \sin(\bar{\varnothing} - \alpha) \text{-----A-2.}$$

If we write (A-2) as

$$\bar{C}' = \bar{R}' \cdot \cos \bar{\varnothing}' ; \bar{S} = \bar{R}' \cdot \sin \bar{\varnothing}' \text{-----A-3}$$

then we have

$$\bar{\varnothing}' = (\bar{\varnothing} - \alpha) \bmod 2\pi ; \bar{R}' = \bar{R} \text{-----A-4.}$$

Hence Equation 3-2 is satisfied. Also from (3-4) and (3-5) we have

$$\sum \sin(\varnothing_i - \bar{\varnothing}) = 0 \text{-----A-5}$$

which corresponds to the equation in the linear case  $\sum(X_i - \bar{X}) = 0$ , i.e. the sum of deviation about the mean vanishes.

### Circular Standard Deviation

. We have seen that  $S_0$  lies in the range  $(0, 1)$ . To relate  $S_0$  to the standard deviation on the line, it is natural to use the following result for the wrapped normal distribution.

$1 - S_0 = e^{-\sigma^2/2}$ . We can then define the circular standard deviation as

$$\sigma = \sqrt{-2 \cdot \log_e(1 - S_0)} \text{-----A-6}$$

The range of  $\sigma$  is of course  $(0, \infty)$ . For small  $S_0$ , Equation A-6 reduces to  $\sigma = (2 \cdot S_0)^{1/2}$ . a transformation first suggested





by Von Mises, 1918).

















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